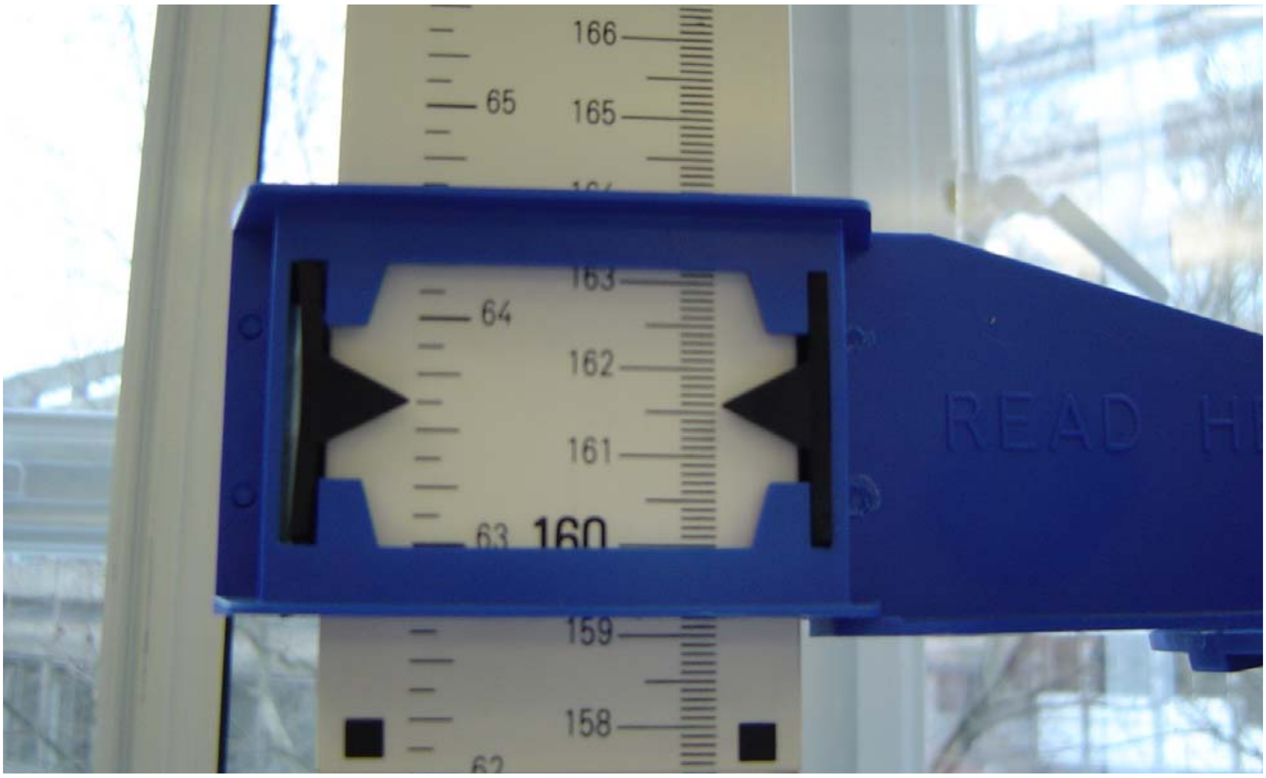
Skills for Life
Support Programme

Resource pack for Level 3 preparation programmes for entry to numeracy teacher training



The Skills for Life Support Programme is delivered on behalf of the Learning and Skills Improvement Service by CfBT Education Trust and partners

CfBT Education Trust
60 Queens Road
Reading
RG1 4BS

T: 0118 902 1920
F: 0845 838 1207
E: sflenquiries@cfbt.com
W: www.excellencegateway.org.uk/sflsp

Acknowledgements

This pack is the result of input and work from a range of individuals including the participants at the three events run by the Skills for Life Support Programme in 2010 and the members of CfBT, LLU+ at London South Bank University, LSN, NIACE and NRDC. The authors wish to thank these individuals for their input into this resource.

Contents

Chapter 1
Introduction and rationale

1. Introduction and rationale

Introduction

This pack has been produced as part of the LSIS Skills for Life Support Programme. The pack is based on a level 3 Advanced Numeracy for Teachers that LLU+ has been running since 2005. The course was a development from an earlier preparatory course for the FENTO level 4 Numeracy Subject Specialists course and is currently run as a preparatory course for the Additional Diploma and Diploma for Teaching in the Lifelong Learning Sector numeracy programmes. Initially the accreditation was through the Open College Network London Region (OCNLR) and it is currently accredited through London South Bank University (LSBU).
The materials include brief instructions for users. See section 6 for more detailed teacher notes.

Rationale

It has become established practice to set all numeracy teaching and learning in an appropriate context and, and the context here is numeracy teacher education and training. With this in mind, the course was developed to include a number of elements that teacher trainers might consider when running their own programmes. These elements are important for all levels of numeracy training and it was thought helpful to identify these issues at the preparation stage.

The elements are:

Personal mathematics skills: the course has the development of personal mathematics skills as its primary focus. Many reports on adult numeracy provision voice concerns about the quality and competence of those who support the numeracy development of the workforce. It was believed that numeracy teachers needed to have maths skills at a higher level than the previously accepted norms. An NRDC literature review (Morton et al (2006)) pointed out that, for schools, the links 'between teachers' subject knowledge, their practices and student achievement' (p23) are not clear. They cite the work of Gess-Newsome 1999 and argued that 'teachers do need to have deep, well-organised and flexible content knowledge in order to represent knowledge to students and select from different instructional approaches' (p23). They concluded that 'teachers having low levels of subject matter knowledge often teach for factual knowledge, involve students in lessons primarily through low-level questions, are bound to content and course structures found in textbooks, have difficulty identifying student misconceptions, and decrease student opportunities to freely explore content either through manipulatives or active discussion' (Gess-Newsome 1999).

NRDC research by Cara and de Coulon (2008) on Skills for Life teachers' qualifications and learner progress in numeracy also found that subject knowledge is of 'prime importance'. They found that learners make more progress when their teachers are qualified to at least Level 3, i.e. A-level or equivalent in mathematics. No effects on improvements were detected for numeracy teachers holding qualifications at Level 2.

Self-assessment: the first session includes carousel of self-assessment tasks so that trainees can identify their own strengths and areas for development. This provides an opportunity for the trainer to observe trainees' on task, and also to identify individual support needs. The resulting needs analysis is useful for planning individual support and drawing up personal development plans. Additionally, the self-assessment tasks give participants an overview of what they will be studying on the course.

History of mathematics: the course content includes an element of the history of mathematics. This gives participants an opportunity to place mathematical problems in a historical context and emphasises the way mathematics can change over time. Variations in development in different geographical locations and cultures help to give participants a broader and more rounded view of mathematics and its place in society. For some, engagement with the history of mathematics is a motivation for learning and doing. In the context of a multicultural classroom, history of maths activities are a practical approach to inclusiveness and valuing diversity.

Profound understanding of fundamental mathematics: One approach used on the course is to promote a 'profound understanding of fundamental mathematics'. The approach is inspired by Liping Ma's book "Knowing and Teaching elementary mathematics" (Ma 1999). In the book, Liping Ma finds from research a correlation between Chinese students performance in mathematics and Chinese teachers understanding of fundamental mathematics, when compared with that of US counterparts, i.e. Chinese teachers appeared to have both a clear understanding of the concepts as well as the ability to teach them. While this work was an investigation of 'elementary school' mathematics teaching, there are lessons to be learned for all phases of education. The approach also offers the opportunity to provide links across different mathematical areas; something which Chinese teachers were found to do to a much greater extent than US teachers. The need to make connections is further supported by research by NRDC on what makes for effective teaching (Coben et al 2007) .

Sophisticated use of elementary mathematics: Sessions on data handling include a look at how individuals perform quite complex analysis using relatively simple statistical techniques. This approach has been inspired by a paper by Margaret Brown and others (Brown et al (2006)) called "Functional mathematics and its assessment" and is closely linked to the development of the entry criteria for initial teacher training, from which the entry assessments that are attached to this course were developed. It has been argued that in today's workplace, the usage of mathematics is more likely to be a sophisticated application of basic mathematical calculations supported by technology, and decision making facilitated by data analysis tools.

Errors and misconceptions: The topic of learner errors and misconceptions runs like a thread through the course. Its inclusion provides an opportunity for participants to review their own mathematical skills and knowledge and also provides an opportunity to address pedagogy on a course that has personal skills as its primary focus, as a range of alternative teaching and learning strategies and techniques can be explored. Research supporting the Standards Unit resource "Improving Learning in Mathematics (Malcolm Swan, 2005)" argues that 'teaching mathematics is effective when it… exposes and discusses common misconceptions'. In "Numeracy, Mathematics and everything: is the answer 42? (Kaye and Chanda 2008)", the authors report that 'non-specialist teachers and support staff now involved in embedding/integrating numeracy are ill-prepared to meet the needs of learners' habitual errors or misconceptions, and that many are hampered also by the fact that they may themselves be operating on the margins of their own competence in numeracy.'

Other resources

The pack is not intended to be comprehensive but indicative of a number of approaches that could be taken in training, or indeed in teaching at any level. In addition to the resources in the pack, on the actual course a number of other more traditional worksheets and texts were used to enable trainees to practice the skills identified in the pack. Appropriate material can be found in a range of Higher Level GCSE textbooks or online.

It should also be noted that the entry criteria do not prescribe a particular set of content to be covered. This means that some trainers may choose to include other topics or processes that they feel are important or could be used as extension material.

Suggestions for topics include:

- Finding arcs and areas of parts of circles (segments and sectors)
- Finding volumes and surface areas of cylinders, prisms and pyramids
- Finding lengths and areas of triangles using sine and cosine rules

Suggestions for processes include:

- What's involved in solving mathematical problems?
- Thinking and reasoning
- Language for interpreting and presenting data
- Using data collation and analysis tools

A project run by the Skills for Life Network in the North East region developed a similar preparation programme although using the personal skills components of the City and Guilds 9484 Level 3 Certificate in Adult Numeracy Subject Support.

Using ICT as a learning resource

Along with the approaches suggested above, the use of a range of ICT could be encouraged, as this reflects the reality of the world beyond the classroom. Developing calculator skills could build confidence in maths usage.. This could include understanding of the differences between basic and scientific calculators, mobile phone calculators and those on computers. Course participants could also be encouraged to use specialist software and more generic applications such as spreadsheets.

References

- Brown M., Coben D., Hodgen J., Stevenson I., and Venkatakrishnan H. (2006) 'Functional Mathematics and its Assessment', in Hewitt, D. (2006) *Proceedings of the British Society for Research into Learning Mathematics.* 26(1) February 2006.

- Cara, O. & de Coulon, A. (2008) *Skills for Life teachers' qualifications and their learners' progress in adult numeracy. Analysis of data from the Skills For Life Teacher and Learner studies.* London : NRDC.

- Chanda, N (2008) unpublished reports for QIA (Quality Improvement Agency).

- Coben, D. Brown, M., Rhodes, V. Swain, J., Ananiandou, K., Brown, P., Ashton, J., Holder, D., Lowe, S., Magee, C., Nieduszynska, S. and Storey, V. (2007) *Effective teaching and learning numeracy.* London : NRDC

- Gess-Newsome, J. (1999). 'Secondary teachers' knowledge and beliefs about subject matter and their impact on instruction', in Gess-Newsome, J., & Lederman, N.G. (1999) *Examining pedagogical content knowledge.* Dordrecht: Kluwer.

- Kaye, D. & Chanda, N. (2009) 'Numeracy, Mathematics and Everything; is the answer 42? Developments in Adult numeracy practice and professional development in the UK during 2007 and 2008',.in Safford-Ramus, K. (ed.) (2009) *A Declaration of Numeracy: Empowering Adults through Mathematics Education, Proceedings of the 15th International Conference of Adults Learning Mathematics (ALM).* Lancaster, PA: DES*tech* Publications.

- Ma, L. (1999) *Knowing and Teaching Elementary Mathematics.* Lawrence Erlbaum Associates, Publishers.

- Morton, T. McGuire, T. and Baynham, M. (2006) *A literature review of research on teacher education in adult literacy, numeracy and ESOL.* London : NRDC.

- Swan, M. (2005) *Improving learning in mathematics: challenges and strategies.* University of Nottingham, Standards Unit available on NCETM website: https://www.ncetm.org.uk/resources/1442

Chapter 2
Self assessment, number and geometry

2.1 Level 3 Self Assessment Carousel Activity

Plan

1 hour 10 mins	**Carousel of advanced numeracy activities** See list of activities and instructions on next page. 1 or 2 activities to be set out per table, tables in caberet style around the room (if short of space, activities can be placed face down in the centre of each table until required). Participants move from one table to another to try out the activities, and use review sheet to record comments. Trainer to intervene and provide support with activities where requested. Trainer should emphasise that participants are not expected to be able to do all the activities – the idea is for them to assess which ones they are confident on, which they need to brush up on and which they need a lot of help with – and the Response sheet is to record this.	Numeracy activity sheets (a – i) Carousel Review sheet
15 mins	**Discussion on the topics and activities completed** For example, how did the exercise feel? Which activities were 'doable' and which were not? What do participants feel they have learned about their personal numeracy skills? What about the methods used – cards, worksheets etc? Did that make a difference? How much did people work together? Did it help? What about the Handy Hints? What implications does this have for teaching? If the group is large, these points could be discussed in small groups and then summarised as feedback to the whole group. Trainer to go through some 'solutions' if requested	Solutions handout (H1)

Trainer notes for Carousel activities

- Explain to participants that there is a choice of 9 Level 3 numeracy activities laid out on tables for them to try out in small groups of two or three.
- Participants work in small groups at each activity in turn.
- Once one activity is completed participants move on to the next one until they have tried all the activities.
- Numeracy teacher trainers should work with different groups in order to get a feel for how participants are managing with the level of the work. Participants make a note of topics they feel they would like further practice in.

List of Carousel Activities

GCSE Probability questions (Activity A)

Worksheet Exercise – pile of worksheets to be left on table

What is Pi? (Activity B)

A4 question sheet - pile of question sheets to be left on table for individuals complete

Simultaneous Equations (Activity C)

Two different word problems to be solved as simultaneous equations
Solutions to simultaneous equations are written on cue cards – one set – left on table.
Cards are then to be sorted and placed in the correct order to solve both equations.
Once equations have been solved, cards are mixed up again for the next group to try.

Straight Line Graphs (Activity D)

Match four equations with four straight line graphs
One set of cards with equations and cards with graphs to be left on table

Standard Form (Activity E)

A variety of number cards are written normally and in standard form on separate cards – 1 set to be left on table. The cards are mixed together.
Match standard form number cards with correct normal number card.

Cumulative Frequency Curve Worksheet (Activity F)

Pile of worksheets to be left on table
Interpret a cumulative frequency graph to estimate the median, lower quartile, upper quartile and interquartile range.

Negative indices (Activity G)

Match negative indices card with equivalent fraction card e.g. $\frac{1}{4} = 4^{-1}$
One set of cards to be left on table

Percentages (Activity H)

Three percentage problems to solve
A3 sheets with space for multiple responses – one copy of each to be left on table with A4 instruction sheet
Numeracy teacher trainer to encourage participants to compare different methods for calculating percentages

Shapes (Activity I)

Match cards with names of shapes to cards with their descriptions – one set to be left on table

Carousel Activity A (1 copy per participant)

Probability

1. **A bag contains 6 green and 4 blue discs. Two discs are taken out without replacement and their colour noted.**
What is the probability that the two discs are
a) both blue?
b) the same colour?

Example of how to do part (a)
a) $^{4}/_{10} \times {}^{3}/_{9} = {}^{12}/_{90}$

Now try part b)

2. **Ceri has 20 tins of soup. 7 are tomato, 9 are potato, 4 are carrot. She has removed the labels for a competition.**

Two tins are chosen at random.
a) What is the probability that they are both carrot?

If the first tin chosen is potato
b) What is the probability that the next tin is tomato?

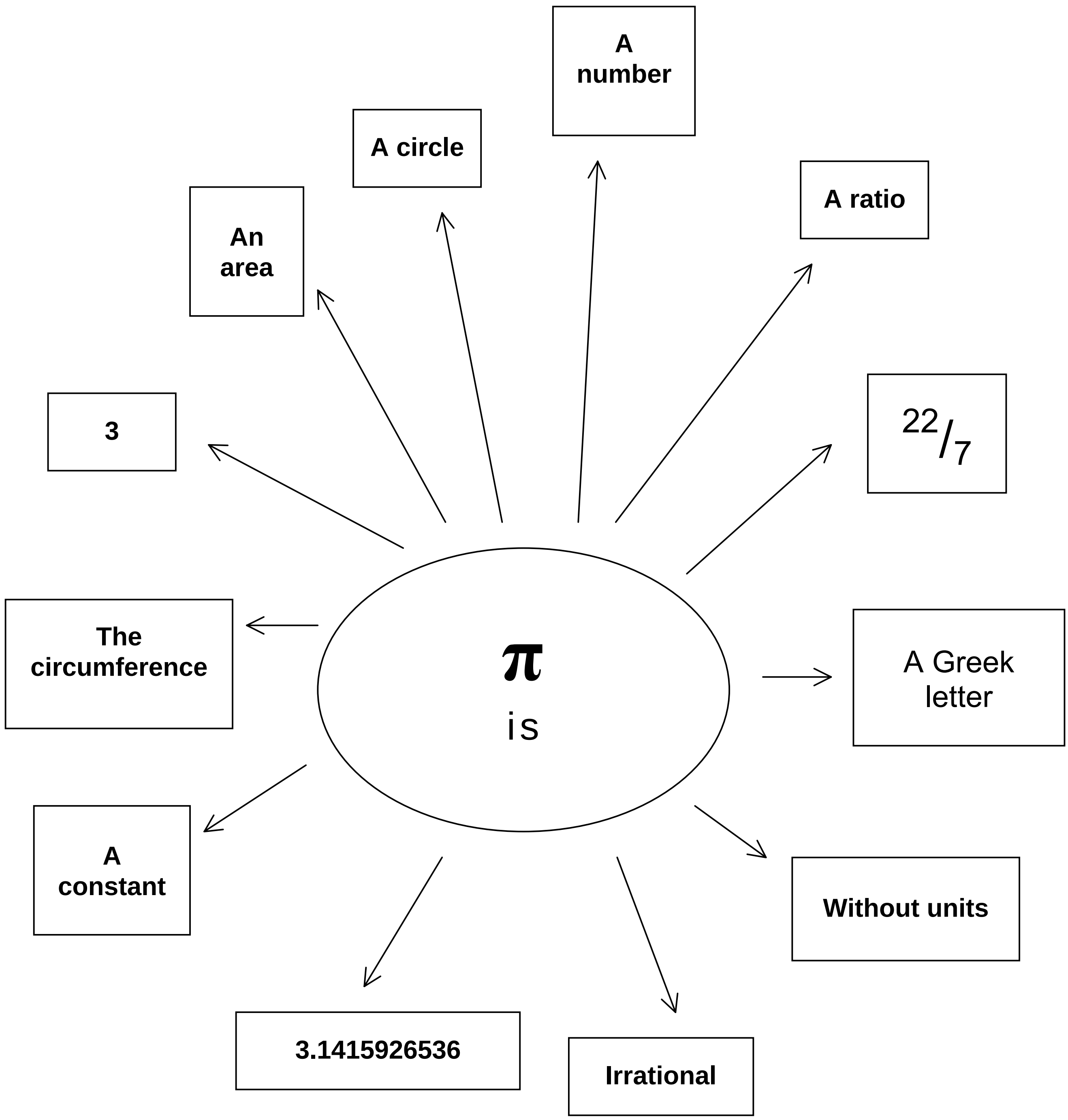
c) What is the probability that the next tin will be something different?

Ceri chooses three tins from the original 20.

d) What is the probability that they are three different types of soup?

Carousel Activity B (1 copy per participant)

What is pi?



Which of these descriptions of π is: **accurate, true, approximate or false**?

Carousel Activity C1 (to be laminated and cut into cards)

2a + 3c = 748	3a + 4c = 1056
6a + 9c = 2244 6a + 8c = 2112- c = 32	2a + 3 × 132 = 748
2a = 748 – 396	2a = 352
352 + 2 × 132	352 + 264
Answer £616	

Question
A holiday for 2 adults and three children costs £748.
The same holiday for 3 adults and 4 children costs £1056.
What would be the cost of the holiday for 2 adults and 2 children?

Question
The ages of a man and his daughter total 27 years.
In 4 years time, the father will be four times as old as his daughter.
How old are they both now?

Carousel Activity C2 (to be laminated and cut into cards)

f + d = 27	f+ 4 = 4(d + 4)
f +4 = 4d + 16	f = 4d + 12
4d + d + 12 = 27	5d = 15
d = 15 ÷ 5	d = 3
f + 3 = 27	f = 27 – 3
f = 24	**Answer** The father is 24 years old. The daughter is 3 years old

Carousel Activity C3

Instructions

Simultaneous equations

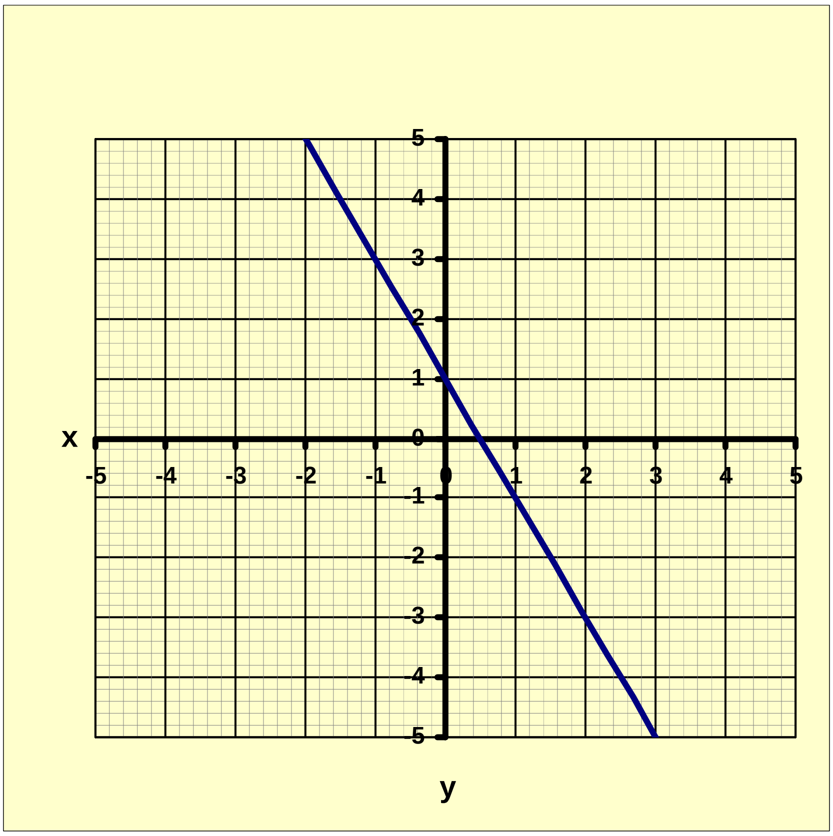
Here are two simultaneous equations problems.

Sort out the equations belonging to each problem.

Put the equations in order to solve the problems.

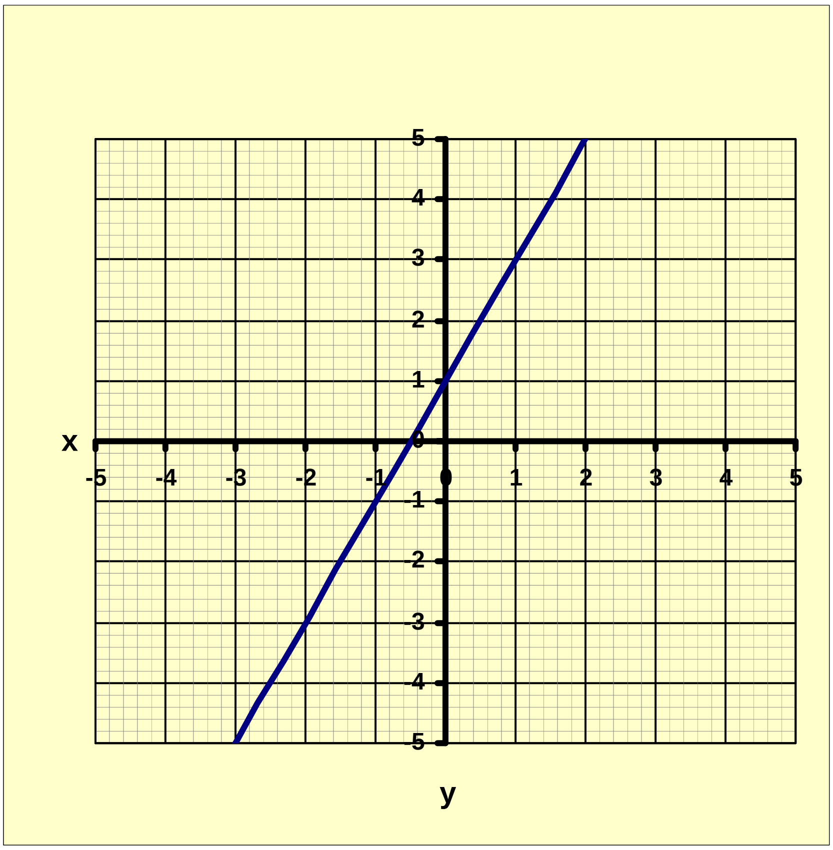
Carousel Activity D1

Card with graph to be laminated and cut out (single copy)



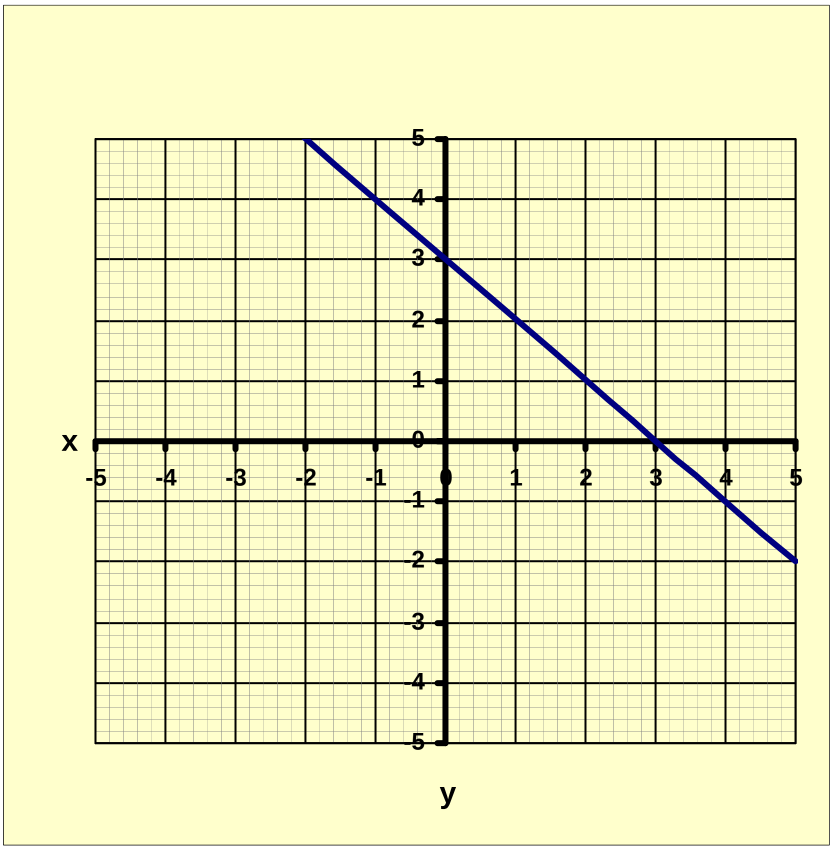
Carousel Activity D2

Card with graph to be laminated and cut out (single copy)



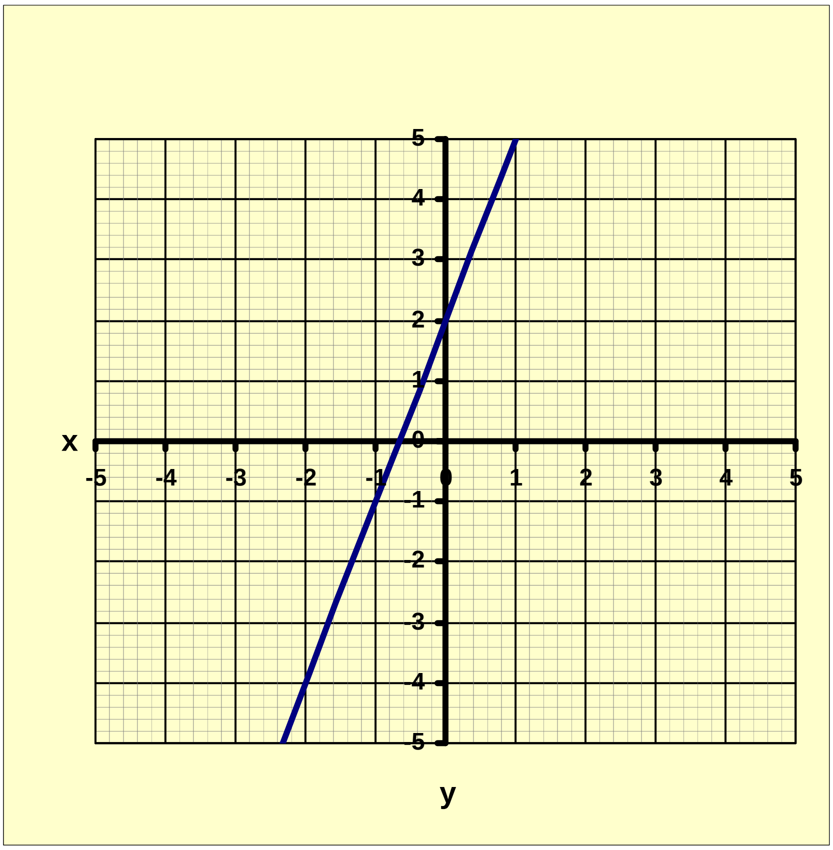
Carousel Activity D3

Card with graph to be laminated and cut out (single copy)



Carousel Activity D4

Card with graph to be laminated and cut out (single copy)



Carousel Activity D5

Cards to be laminated and cut out (single copy of each)

$y = 2x+1$
$y = 3x + 2$
$y = 3 - x$
$2x + y = 1$

Carousel Activity D6

Instructions

Straight line graphs

Match the equations to the graphs.

Carousel Activity E1

1 copy A3 size, to be laminated and cut up as cards

2567	2.567×10^3
2000000000	2.0×10^9
2678589	2.678589×10^6
589.32	5.8932×10^2
8306000	8.306×10^6
1769000000	1.769×10^9
28000	2.8×10^4
589.32	5.8932×10^2
5000000	5.0×10^6
58.9	5.89×10^1
83067	8.3067×10^4
26745.9	2.67459×10^4
35672080	3.567208×10^7
0.589	5.89×10^{-1}
0.0000593	5.93×10^{-5}
0.0028	2.8×10^3
0.00256	2.56×10^{-3}
0.026	2.6×10^{-2}

Carousel Activity E2

Instructions

Standard form

Match the numbers with the equivalent numbers in standard form.

Note: The two types of numbers are colour coded.

Carousel Activity F

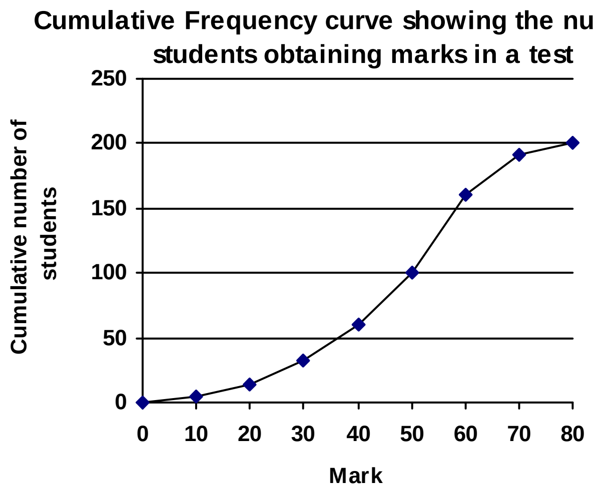
Cumulative Frequency

The marks scored by 200 students in their G.C.S.E. examination are given in the frequency distribution below. Complete the cumulative frequency table for this data.

Frequency distribution / Cumulative Frequency

Mark	Number of students	Mark	Number of students
0 and under 10	0	< 10	0
10 and under 20	4	< 20	4
20 and under 30	8	< 30	
30 and under 40	10	< 40	
40 and under 50	22	< 50	
50 and under 60	70	< 60	
60 and under 70	66	< 70	
70 and under 80	16	< 80	
80 and under 90	4	< 90	

The cumulative frequency curve shown below represents the marks scored by the same 200 students in a '**mock**' G.C.S.E. examination **(i.e. a different examination from that for which the results are listed in the table above)**



(a) From the graph estimate:

(i) The median Answer ....

(ii) The lower quartile Answer ....

(iii) The upper quartile Answer ....

(iv) The interquartile range Answer ....

Carousel Activity G1

Cards to be laminated and cut out (single copy of each)

$\frac{1}{3}$	3^{-1}
$\frac{1}{8}$	8^{-1}
$\frac{1}{4^3}$	4^{-3}
$\frac{1}{8^2}$	$\frac{1}{64}$
$\frac{1}{4}$	4^{-1}
$\frac{1}{2}$	2^{-1}
$\frac{1}{64}$	8^{-2}

Carousel Activity G1

Cards to be laminated and cut out (single copy of each)

2^{-4}	$\frac{1}{2^4}$
$\frac{1}{16}$	

Carousel Activity G2

Instructions

Fractions and negative indices

Match the number with negative indices to the equivalent fractions.

Note: there are some items with three equivalent forms

Carousel Activity H1 (1 copy, A4 size)

Instructions

Percentages

Here are three percentage problems.

Write in your method of solving this sort of problem.

If someone else has used the same method, write a comment about why you favour this method.

Carousel Activity H2 (1 copy, A3 size)

(1) Percentage increase

A friend was earning a salary of £21,920 and after a pay rise in January 2004 is now earning £22,580.
What percentage increase is this?

Carousel Activity H3 (1 copy, A3 size)

(2) “Backwards” Percentage

In a recent sale, there was a 30% reduction on all goods. What was the original price of a jacket sold in the sale for £38.50?

Carousel Activity H4 (1 copy, A3 size)

(3) Adding VAT

Show a non-calculator method to add VAT (17.5%) to a building job priced at £280

Carousel Activity I1

Cards enlarged to A3 and to be laminated and cut out (single copy of each)

Solid with 12 edges	Cuboid
This quadrilateral has all its angles equal but not its sides	Rectangle
A quadrilateral with 4 axes of symmetry	Square
A closed shape made from one curved line and one straight line	Segment
A quadrilateral with only one axis of symmetry	Kite
A solid with four triangular faces	Tetrahedron
Part of a circle cut out by two radii	Sector
A solid with a regular cross section	Prism and/or Cylinder
The regular 3-sided polygon	Equilateral triangle
A solid with a circular cross section	Cylinder and/or cone
No match	Trapezium
No match	Parallelogram
No match	Right-angled triangle
No match	Rhombus

Carousel Activity 2

Instructions

Properties of 2D and 3D shapes

Match the shape to the properties.

There are four shapes with no match

Session 1 - Activity Sheet 2
(1 copy per participant)

CAROUSEL OF MATHS ACTIVITIES – REVIEW SHEET

Please use this sheet to record your responses to each of the mathematics activities you tried. Your responses will help the teacher to plan a programme of mathematics development to suit your individual needs.

Activity	Response – e.g. *easy/need to refresh memory/ could not remember how to do/ need help on this topic /could not understand the task, etc*
Cumulative Frequencies	
Simultaneous Equations	
Fractions & Negative Indices	
Percentages	
Probability	
Standard Form	
Straight Line Graph	
What is π?	
Properties of 2D and 3D Shapes	

What do you think about this approach to assessment?

Handout 1A (1 copy per participant – to be produced as a booklet)

Self-Assessment Carousel of example activities

Solutions and guidance notes

Cumulative Frequencies

Mark	Number of students	Mark	Number of students
0 and under 10	0	< 10	**0**
10 and under 20	4	< 20	**4**
20 and under 30	8	< 30	**12**
30 and under 40	10	< 40	**22**
40 and under 50	22	< 50	**44**
50 and under 60	70	< 60	**114**
60 and under 70	66	< 70	**180**
70 and under 80	16	< 80	**196**
80 and under 90	4	< 90	**200**

(a) From the graph estimate:

(i) The median Answer **50 marks**

(ii) The lower quartile Answer **35 marks**

(iii) The upper quartile Answer **57 marks**

(iv) The interquartile range Answer **22 marks (57 – 35)**

Notes

The median mark is the mark scored by the middle student (50% of the group)

The lower quartile is the mark scored by the student in the middle of the lower half (25% of the group)

The upper quartile is the mark scored by the student in the middle of the upper half (75% of the group)

The interquartile range is the range of marks scored by the middle 50% of the group (between the upper and lower quartiles).

The cumulative frequency table that is completed is different data from that presented in the cumulative frequency graph.

The answers to reading the graph can be ± 3

Handout 1B (1 copy per participant – to be produced as a booklet)

Simultaneous Equations

A holiday for 2 adults and three children costs £748. The same holiday for 3 adults and 4 children costs £1056. What would be the cost of the holiday for 2 adults and 2 children? 2a + 3c = 748 3a + 4c = 1056 6a + 9c = 2244 <u>6a + 8c = 21 12-</u> c = 132 2a + 3 x 132 = 748 2a = 748 – 396 2a = 352 352 + 2 x 132 352 + 264 Answer £616	The ages of a man and his daughter total 27 years. In 4 year's time, the father will be four times as old as his daughter. How old are they both now? f + d = 27 f + 4 = 4(d + 4) f + 4 = 4d + 16 f = 4d + 12 4d + d + 12 = 27 5d = 15 d = 15÷5 d = 3 f + 3 = 27 f = 27 – 3 f = 24 Answer: The father is 24 years old. The daughter is 3 years old

Notes

Identify the two questions and separate out the two sets of equations by the appropriate numbers.

Work with each set separately; identify the equations that set out the first version of the question.

How can you manipulate the equations to isolate one unknown variable?

Once you know the value of one variable, how can you find the value of the other one?

Handout 1C (1 copy per participant – to be produced as a booklet)

Negative Indices/Fractions

$\frac{1}{3} = 3^{-1}$

$\frac{1}{8} = 8^{-1}$

$\frac{1}{4^3} = \frac{1}{64} = 4^{-3}$

$\frac{1}{4} = 4^{-1}$

$\frac{1}{2} = 2^{-1}$

$\frac{1}{64} = \frac{1}{8^2} = 8^{-2}$

$\frac{1}{2^4} = \frac{1}{16} = 2^{-4}$

Notes

Note there are three items that have three equivalent forms (not two)

Remember the negative indices can be read as “divide by”

Handout 1D (1 copy per participant – to be produced as a booklet)

Percentages

(1) ***Percentage Increase***

A friend was earning a salary of £21,920 and after a pay rise in January 2004 is now earning £22,580.
What percentage increase is this?

Using the corrected figure of a new wage of £22,580 gives an answer of approximately 3%.

On the A3 sheet there should be examples of 'working out' like 22,580 – 21,920 = 660

$$\frac{660}{21,920} \times 100$$

The important feature to note is the calculation is made using the actual pay rise (in pounds) divided by the **old** salary

(2) Backwards Percentage

In a recent sale, there was a 30% reduction on all goods. What was the original price of a jacket sold in the sale for £38.50?

The answer is £55.00

The working out should show that if there has been a reduction of 30%, the sale price (of £38.50) is equal to 70%

The answer (£55) is therefore equal to 100%

Examples might be 70% = 38.50

$$100\% = \frac{100}{70} \times \frac{38.50}{1}$$

or 70% = £38.50
10% = £5.50
100% = £55.00

(3) Adding VAT

Show a non-calculator method to add VAT (17.5%) to a building job priced at £280

The answer is VAT is £49 and the total including VAT is £329.

The working out should show that 17.5% can be split into 10% + 5% + 2.5%

Therefore the calculation might look like this

10%	£28
5%	£14
2.5%	£7

Handout 1E (1 copy per participant – to be produced as a booklet)

Probability

1. A bag contains 6 green and 4 blue discs. Two discs are taken out without replacement and their colour noted.
   What is the probability that the two discs are
   a) both blue?
   b) the same colour?

Example of how to do part (a)
a) $^{4}/_{10}$ x $^{3}/_{9}$ = $^{12}/_{90}$

Now try part b)

2. Ceri has 20 tins of soup. 7 are tomato, 9 are potato, 4 are carrot.
   She has removed the labels for a competition.

   Two tins are chosen at random.
   c) What is the probability that they are both carrot?

   If the first tin chosen is potato
   d) What is the probability that the next tin is tomato?

   e) what is the probability that the next tin will be something different?

   Ceri chooses three tins from the original 20.
   f) What is the probability that they are three different types of soup?

Part 1a is given on the sheet as an example of something and something else happening, which is calculated by multiplying the separate probabilities.

The first blue is 4 out of 10, which is **not** replaced. The second blue is 3 out of 9. These are multiplied to give the combined probability.

4/10 x 3/9 = 12/90 (or 2 out of 15)

(1b) (6/10 x 5/9) + (4/10 x 3/9) = 30/90 + 12/90 = 42/90 = 7/15

(2a) The two tins are chosen one at a time. The first tin is **not** replaced before taking out the second.

4/20 x 3/19 = 12/380 = 3/95

(2b) This is a conditional probability question. Given that the first tin chosen is potato then there are 19 tins remaining and 7 are tomato.

7/19

(2c) Given that the first tin chosen is potato then 19 are remaining 11 of these are tomato or carrot.

11/19

Handout 1E (1 copy per participant – to be produced as a booklet)

(2d) There are six permutations possible for picking different soups. The probability of each is the same - see below (63/170).

The probability of picking tomato then potato then carrot, i.e. TPC =7/20 x 9/19 x 4/18 = 252/6840 = 63/1710

The probability of picking tomato then carrot then potato, i.e. TCP = 7/20 x 4/19 x 9/18 = 63/1710

Probability of CTP = 4/20 x 7/19 x 9/18 = 63/1710

Probability of CPT = 4/20 x 9/19 x 7/18 = 63/1710

Probability of PTC = 9/20 x 7/19 x 4/18 = 63/1710

Probability of PCT = 9/20 x 4/18 x 7/18 = 63/1710

You want the probability that the first permutation happens, or the second, or the third, or the fourth, or the fifth, or the sixth. So you need to add all the probabilities above together.

Since they are all the same, this is the same as multiplying one of them by 6

So **the probability that they are three different types of soup is** 6 x 63/1710

Handout 1F (1 copy per participant – to be produced as a booklet)

Shapes

Solid with 12 edges	Cuboid
This quadrilateral has all its angles equal but not its sides	Rectangle
A quadrilateral with 4 axes of symmetry	Square
A closed shape made from one curved line and one straight line	Segment
A quadrilateral with only one axis of symmetry	Kite
A solid with four triangular faces	Tetrahedron
Part of a circle cut out by two radii	Sector
A solid with a regular cross section	Prism and/or Cylinder
The regular 3-sided polygon	Equilateral triangle
A solid with a circular cross section	Cylinder and/or cone
No match	Trapezium
No match	Parallelogram
No match	Right-angled triangle
No match	Rhombus

It is useful if there are some shapes and solids to demonstrate some of these descriptions

A cone is included in the final definition, as it does not say a ‘regular’ circular cross section

A cylinder is included in the prism definition as it is a type of prism, though the description given is general

Note the definition of the rhombus is a reverse of that for the rectangle i.e. it is a quadrilateral with all its sides equal, but not its angles

Handout 1G (1 copy per participant – to be produced as a booklet)

Descriptions of π

The following are true

π is a constant

π is a ratio

π is a number

π is a Greek letter

π is irrational

π is without units

The following are approximate descriptions of π

3.1415926536

3

$^{22}/_{7}$

The following are false descriptions of π

Circumference

Area

Circle

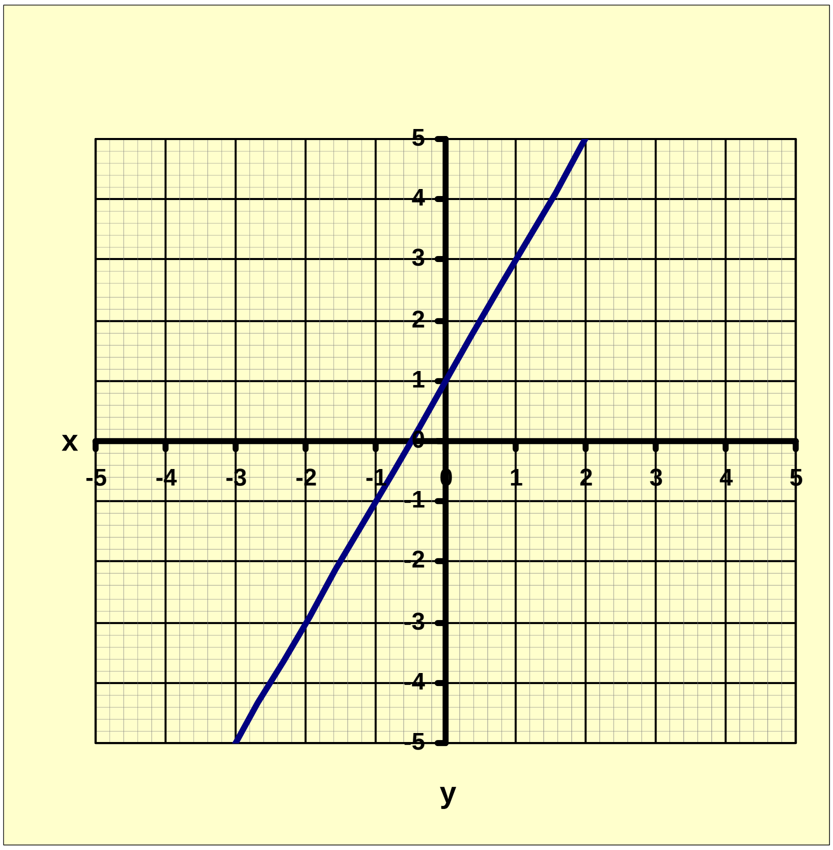
Handout 1H (1 copy per participant – to be produced as a booklet)

Straight line Graphs

Take the co-ordinates of a point which the line goes through, e.g. (0,1), and see if the co-ordinates fit the selected equation

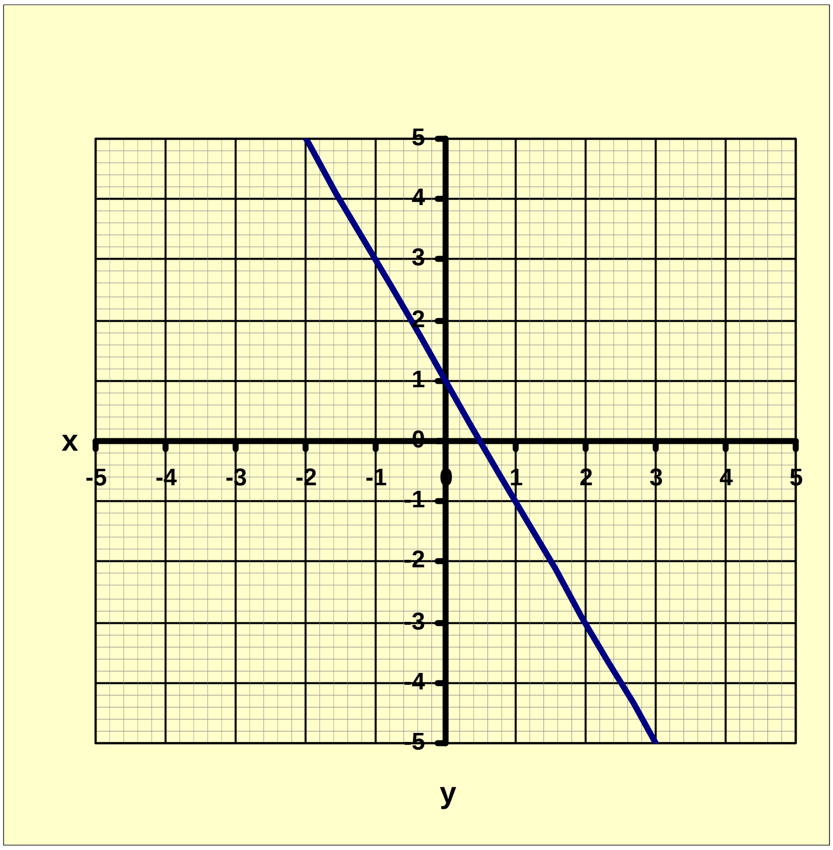
So if x = 0, then in the equation below, 2x + 1 = 2 x 0 + 1 = 1, which is equal to the y co-ordinate, so it must be the right equation for the graph.

y = 2x + 1



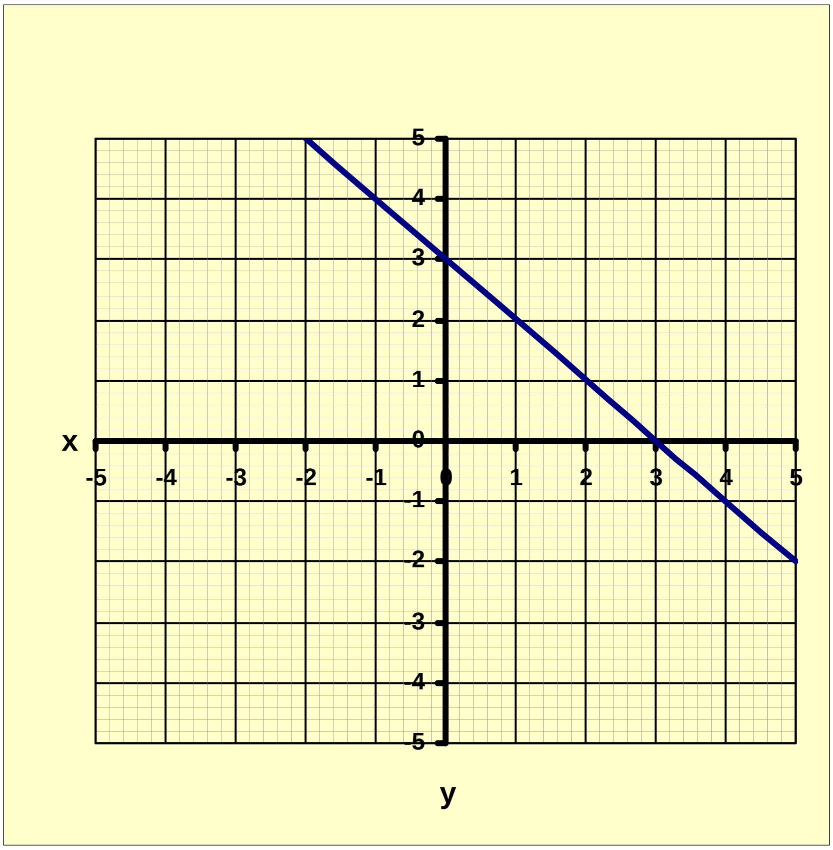
Handout 1l (1 copy per participant – to be produced as a booklet)

2x +y =1



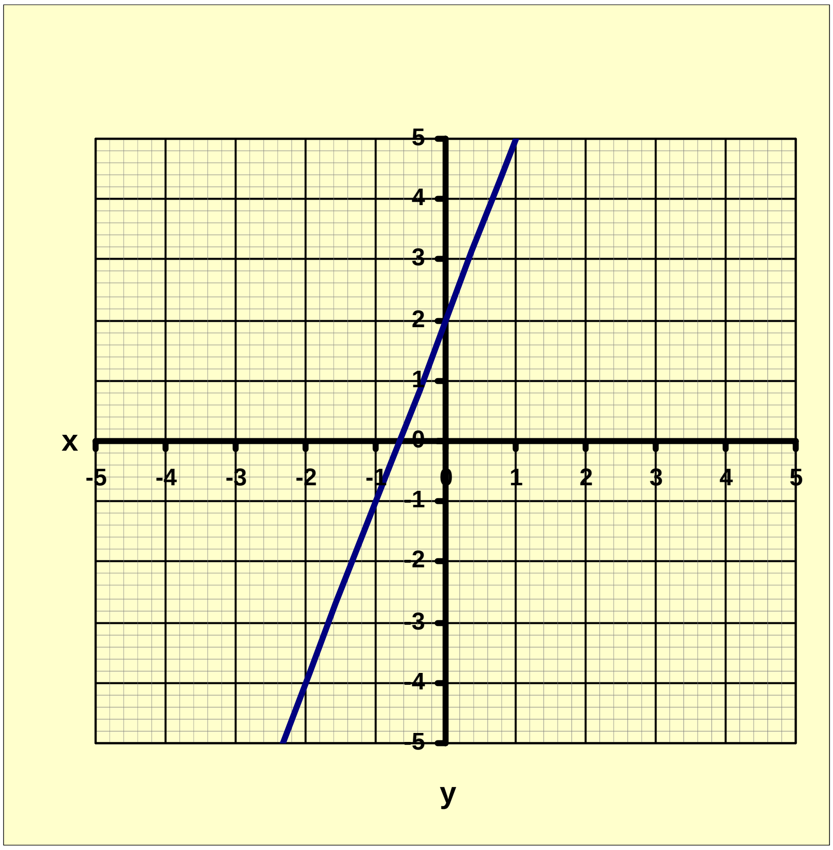
Handout 1J (1 copy per participant – to be produced as a booklet)

y = 3 - x



Handout 1K (1 copy per participant – to be produced as a booklet)

y = 3x + 2



Handout 1L (1 copy per participant – to be produced as a booklet)

Standard form

2567	2.567×10^3
2000000000	2.0×10^9
2678589	2.678589×10^6
589.32	5.8932×10^2
8306000	8.306×10^6
1769000000	1.769×10^9
28000	2.8×10^4
589.32	5.8932×10^2
5000000	5.0×10^6
58.9	5.89×10^1
83067	8.3067×10^4
26745.9	2.67459×10^4
35672080	3.567208×10^7
0.589	5.89×10^{-1}
0.0000593	5.93×10^{-5}
0.0028	2.8×10^3
0.00256	2.56×10^{-3}
0.026	2.6×10^{-2}

2.2 Checklist of Level 3 Personal Skills

Name:	Key : ☺= can do, 😐= needs more practice, ☹= needs lots of work

Please tick the most appropriate box.

Curriculum description	☺	😐	☹	Comments
Number and Numeric Systems				
The ability to read, write and represent numbers and numerical relationships in a wide variety of ways.				
The place value system, expanded notation and practical application of number theory (concepts of factors, multiples, order relations and order of operations).				
Computing with whole numbers, integers, fractions, decimals and percentages using appropriate algorithms and a variety of techniques (mental computation, pencil-and-paper, calculator and computer methods).				
Converting between percentages, fractions and decimals. Manipulating fractions, decimals and percentages				
Calculating proportional change.				
Developing skills necessary to estimate sums, differences, products and quotients.				

Curriculum description	☺	😐	☹	Comments
Manipulating and calculating squares and square roots and numbers expressed in standard notation.				
Understanding mathematical information presented as standard compound measures and units, common powers and roots.				
Understanding and creating two and three part ratios, calculating ratios and reducing ratios to lower terms				
Interpreting representations of scale such as drawings and maps to establish actual dimensions				
Understanding how to convert measurements between common scales				
Creating scale drawings using appropriate measurements.				
Measurement, Geometry and Spatial Awareness				
Using exact and estimated measurements to describe and compare phenomena.				
Calculating perimeter and area measurements (squares, rectangles, and parallelograms including rhombus, trapezoids, and triangles).				
Establishing measurements (circumference, area, radius and diameter) for a circle from known variables				
Measuring attributes of physical objects (length, time, temperature, capacity, weight, mass, area, volume and angle).				
Understanding and applying geometric properties and relationships.				

Curriculum description	☺	😐	☹	Comments
Classifying and comparing geometric figures (e.g. triangles, quadrilaterals)				
Exploring transformations of geometric figures				
Representing and solving problems using geometry (e.g. height of a building)				
Applying the use of appropriate technologies to the study of geometry and spatial sense				
Working with Formulae				
Choosing and applying appropriate formulas to solve problems involving perimeter, area, and volume.				
Rearranging basic algebraic expressions by collecting terms, expanding brackets, extracting common factors, and finding the value of an unknown				
Solving simultaneous linear equations with two variables using appropriate algebraic techniques				
Rearranging quadratic functions and trigonometric equations				
Graphic calculators/graph plotting software to: • plot graphs of data pairs • plot graphs of functions • use function facilities				
Statistics				
Basic sampling techniques and sampling distributions				
Tabulation techniques, and creating and interpreting charts and graphs. This should involve constructing, reading and interpreting tables, charts and graphs.				

Curriculum description	☺	😐	☹	**Comments**
Probability - basic expressions and forms of representation of chance (e.g. tree diagrams) and the associated terminology				
Measures of location - mean, median and mode				
Measures of spread - upper and lower quartiles, percentiles; and standard deviation				
Statistical diagrams would typically cover histograms, pie charts, frequency polygons, cumulative frequency diagrams, scatter diagrams.				
For correlation- understanding ideas of: • positive correlation, negative; • correlation and no correlation; • correlation co-efficient.				
For regression- understanding ideas of: • equation for line of best fit; • drawing regression lines on scatter plots; • assessing how well regression line fits the observed data.				
Using calculators to find: mean, standard deviation, sum of values, correlation coefficient, linear regression coefficients.				
Using spreadsheets to record data in tables; calculate values from data; plot graphs; draw statistical diagrams; calculate statistical measures.				

2.3 Level 3 Preparatory Programme
Group Summary Checklist

Number and Numeric Systems

Participants' initials																	
Curriculum description *The ability to read, write and represent numbers and numerical relationships in a wide variety of ways.*																	
The place value system, expanded notation and practical application of number theory (concepts of factors, multiples, order relations and order of operations).																	
Computing with whole numbers, integers, fractions, decimals and percentages using appropriate algorithms and a variety of techniques (mental computation, pencil-and-paper, calculator and computer methods)																	
Converting between percentages, fractions and decimals. Manipulating fractions, decimals and percentages																	
Calculating proportional change																	
Developing skills necessary to estimate sums, differences, products and quotients																	

Manipulating and calculating squares and square roots and numbers expressed in standard notation.																	
Understanding mathematical information presented as standard compound measures and units, common powers and roots.																	
Understanding and creating two and three part ratios, calculating ratios and reducing ratios to lower terms																	
Interpreting representations of scale such as drawings and maps to establish actual dimensions																	
Understanding how to convert measurements between common scales																	
Creating scale drawings using appropriate measurements.																	

Measurement, Geometry and Spatial Awareness

Participants' initials																	
Curriculum description *Using exact and estimated measurements to describe and compare phenomena.*																	
Calculating perimeter and area measurements (squares, rectangles, and parallelograms including rhombi, trapezoids, and triangles).																	
Establishing measurements (circumference, area, radius and diameter) for a circle from known variables.																	
Measuring attributes of physical objects (length, time, temperature, capacity, weight, mass, area, volume and angle).																	
Understanding and applying geometric properties and relationships.																	
Classifying and comparing geometric figures (e.g. triangles, quadrilaterals).																	
Exploring transformations of geometric figures.																	
Representing and solving problems using geometry (e.g. height of a building).																	
Applying the use of appropriate technologies to the study of geometry and spatial sense.																	

Working with formulae

Participants' initials																	
Curriculum description *Choosing and applying appropriate formulas to solve problems involving perimeter, area and volume.*																	
Rearranging basic algebraic expressions by collecting terms, expanding brackets and extracting common factors and finding the value of an unknown.																	
Solving simultaneous linear equations with two variables using appropriate algebraic techniques																	
Rearranging quadratic functions and trigonometric equations																	
Graphic calculators/graph plotting software to: • *plot graphs of data pairs* • *plot graphs of functions* • *use function facilities*																	

Statistics

Participants' initials																	
Curriculum description *Basic sampling techniques and sampling distributions*																	
Tabulation techniques and creating and interpreting charts and graphs. This should involve constructing, reading and interpreting tables, charts and graphs.																	
Probability - basic expressions and forms of representation of chance (e.g. tree diagrams) and the associated terminology																	
Measures of location - mean, median and mode																	
Measures of spread - upper and lower quartiles, percentiles and standard deviation																	
Statistical diagrams: histograms, pie charts, frequency polygons, cumulative frequency diagrams, scatter diagrams.																	
For correlation understanding ideas of: • *positive correlation, negative correlation and no correlation* • *correlation co-efficient*																	

For regression understanding ideas of: • *equation for line of best fit* • *drawing regression lines on scatter plots* • *assessing how well regression line fits the observed data*																	
Using calculators to find: mean, standard deviation, sum of values, correlation coefficient, linear regression coefficients.																	
Using spreadsheets to record data in tables; calculate values from data; plot graphs; draw statistical diagrams; calculate statistical measures.																	

2.4 Number systems and place value

The Hindu-Arabic number system

The number system we use today was first used by Hindu mathematicians around 200BC. The Hindus had a symbol for the numbers one to nine. In the 9th century they invented a symbol called “sunya,” meaning empty, which we now call zero. It is a decimal system which uses place value.

The system was introduced to Europe by Arab traders around the 8th century.

Mathematicians regard the Hindu-Arabic system as one of the world's greatest inventions. Its greatness lies in the principle of place value and in the use of zero. These two ideas make it easy to represent numbers and to perform mathematical operations that would be difficult with any other kind of system.

The Babylonian number system

The Babylonians lived in Mesopotamia, which is between the Tigris and Euphrates rivers. They began a numbering system about 5,000 years ago. It is one of the oldest numbering systems.

It was a base 60 (sexigesimal) rather than a base ten (decimal) system as we use today. It is also credited as being the first known place-value numeral system, in which the value of a particular digit depends both on the digit itself and its position within the number. This was an extremely important development, because prior to place value systems people were obliged to use unique symbols to represent each power of a base (ten, one hundred, one thousand, and so forth), making even basic calculations unwieldy.

Rather than using ten symbols as we use today, the Babylonians only used two symbols.

- To represent 1  and to represent 10

However, they had no symbol for zero; instead they left a space to indicate a place without value.

1	11	21	31	41	51
2	12	22	32	42	52
3	13	23	33	43	53
4	14	24	34	44	54
5	15	25	35	45	55
6	16	26	36	46	56
7	17	27	37	47	57
8	18	28	38	48	58
9	19	29	39	49	59
10	20	30	40	50	60

Notice how the number 60 is represented and compare it to how 1 is represented.

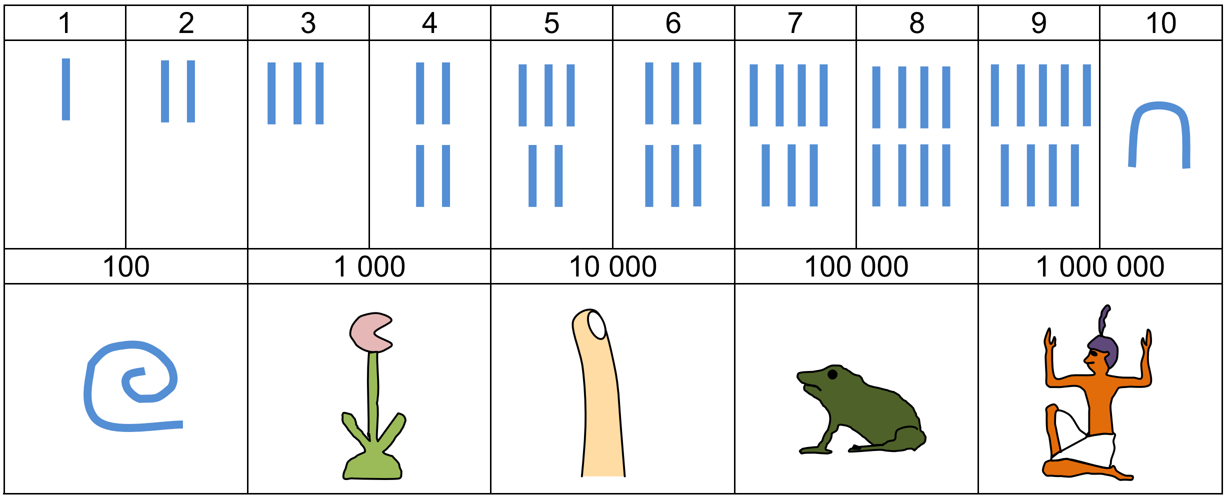
We still use this base 60 system today. Can you think of any examples?

The Egyptian number system

The Egyptian number system dates from about 3000 BC. The Egyptians used a decimal number system that was changed into hieroglyphic writing. It has been found on the writings on the stones of monument walls of ancient time. Numbers have also been found on pottery, limestone plaques, and on the fragile fibres of papyrus.

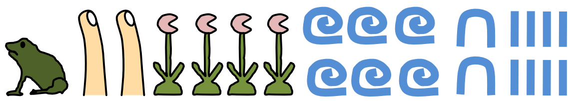
It used seven different symbols.

- 1 is shown by a single stroke.
- 10 is shown by a drawing of a hobble for cattle.
- 100 is represented by a coil of rope.
- 1 000 is a drawing of a lotus plant.
- 10 000 is represented by a finger.
- 100 000 by a tadpole or frog
- 1 000 000 is the figure of a god with arms raised above his head.



Unlike the Babylonian system, it had no place value. However, Egyptian numbers also did not have a symbol to represent zero.

The number 124 628 would be represented



The Roman number system

The Romans were active in trade and commerce, and from the time of learning to write they needed a way to represent numbers. The system they developed lasted many centuries, and still sees some specialised use today. The system used in classical antiquity was slightly modified in the Middle Ages to produce a system used today. It is based on certain letters which are given values as numerals.

Roman Numeral	Number
I	1
V	5
X	10
L	50
C	100
D	500
M	1000

A string of letters means that their values should be added together. For example, XXX = 10 + 10 + 10 = 30, and LXI = 50 + 10 + 1 = 61.

A slight adaptation was introduced to the system in that for smaller values placed before a larger one, we subtract instead of adding. For some instance, IV = 5 - 1 = 4.

982
= 900 + 80 + 2
= CM + LXXX + II
= CMLXXXII.

Roman numerals remained in common use until about the 14th century, when they were replaced by the Hindu-Arabic system.

Examples of their current use include:

- Names of monarchs and Popes are still displayed in Roman numerals, e.g. Elizabeth II, Benedict XVI. These are ordinal numbers; e.g. "II" is pronounced "the second"
- The year and any credits shown at the end of a television show or film
- Some faces of clocks and watches show hours in Roman numerals
- Film series and sequels of novels and video games are often numbered with Roman numerals; this is typically done in emulation of older books
- In formatting I, II, III and i, ii, iii is used as part of the organisational structure
- Recurring grand events, such as the Olympic Games
- Historic events, such as World War II

Can you think of any others?

Websites

http://www.eyelid.co.uk/numbers.htm

http://www.math.wichita.edu/history/topics/num-sys.html#babylonian

http://gwydir.demon.co.uk/jo/numbers/babylon/index.htm

http://en.wikipedia.org/wiki/Roman_numerals

2.5 Number systems

Refer to presentation on website
http://www.excellencegateway.org.uk/283480

Number systems - Fill in the gaps

Babylonian	Egyptian	Roman	Hindu-Arabic
			
		LXXIV	
			19

2.6 Egyptian multiplication

The Egyptians had an interesting way to multiply and they only had to memorise the 2 times table. In fact it is often called 'Doubling' and it works just like it sounds. You take one number and multiply it by 2. This is done repeatedly until you get the answer. Below is an example of what you need to do solve the problem 21 x 22:

Select a set of numbers from this table that add up to 21

21 × 22	
1	22
2	44
4	88
8	176
16	352
32	704
16 + 4 + 1 = **21**	22 + 88 + 352 = **462**

Add together the results of multiplying 22 by these numbers: 22 + 88+ 352 = 462

How to multiply using the Egyptian method

- Choose which number you want to start with - either 21 or 22 will do. This example starts with doubling 22.

- Then set up a little chart. Start with 22 (the number that you are going to double) on the right hand side. On the left hand side start with one. You keep doubling the number on each side. You can stop once your numbers on the left have gone past the original number.

- Now look at the left hand column. You need to find the numbers that add up to the original number, in our case this is 21. In this case, numbers that add up to 21 are 1, 4 and 16.

- Take the corresponding results from the doubling operation and add them together:

  22 + 88 + 352 = 462. This is the product of 21 and 22.

2.7 The commutative, associative and distributive laws

Commutative law

When adding two numbers such as 15 + 18, the result is the same whether you start from the left or the right. The commutative law states that the order in which an addition is done is not important.

$$15 + 18 = 18 + 15$$

This is an important law for your learners to realise as it can help with strategies for mental and paper based addition. If your learner has the question 5 + 63, it is easier to add on 5 to 63 than count on from 5.

Another useful application of the commutative law is seen in multiplication. If you had to answer the following question mentally: What is 28% of £25? You would get the same result if you found 25% of £28. The answer is 7 in both cases.

> Addition and multiplication are commutative, but subtraction and division are not.

Associative law

In this law if you have three numbers to add together you get the same answer whether you start by adding the second and third or the first and second etc.

$$18 + (13 + 7) = (18 + 13) + 7$$

In this example, it might be easier to add 13 and 7 first (using number bonds) than to add 18 and 13. It might help your learners to look at ways numbers can be grouped in order to make it easier to add.

> Addition and multiplication obey the associative law, but subtraction and division do not.

Can you think of a multiplication question which could be used to illustrate the uses of the associative law?

Distributive law

In the standard method, for long multiplication:

345 x 23		3 4 5
		x 2 3
	345 x 3 →	1 0 3 5
	345 x 20 →	6 9 0 0
		7 9 3 5

We have effectively split 23 into 20 and 3 and multiplied 345 by 3, then by 20. This can be represented:

$$a \times (b + c) = (a \times b) + (a \times c) \text{ or}$$
$$345 \times (20 + 3) = (345 \times 20) + (345 \times 3)$$

Using this law, mental multiplication can sometimes be performed easier.

Consider 28 x 6.

$$6 \times (20 + 8) = (6 \times 20) + (6 \times 8) = 120 + 48 = 168$$

> Multiplication and division are said to be *distributive* over addition or subtraction. This means that to multiply or divide the sum (or difference) of two numbers by something you can multiply or divide them separately and then find the sum (or difference) of the results.

Arithmetic Laws

There are a number of laws of arithmetic operation and rules of simplifying arithmetic operation which are useful to understand. Your learners will have to learn the principles of these laws, but not the terminology.

Decide whether you think the following are usually true or false. Substitute numbers for letters to help you to decide if necessary.

What can you say about what you have found? Write down a statement about adding two numbers and adding three numbers.

Laminate on A4 paper and cut out

$a + b = b + a$
$a – b = b – a$
$a \times b = b \times a$
$a \div b = b \div a$

$(a + b) + c = a + (b + c)$
$(a + b) – c = a + (b – c)$
$(a – b) – c = a – (b – c)$
$a \times (b – c) = a \times b - c$
$a \times (b + c) = a \times b + a \times c$
$a \div (b + c) = a \div b + a \div c$
$(b + c) \div a = b \div a + c \div a$

Answers

1.	a + b = b + a	true
2.	a – b = b – a	false
3.	a x b = b x a	true
4.	a ÷ b = b ÷ a	false
5.	(a + b) + c = a + (b + c)	true
6.	(a + b) – c = a + (b – c)	true
7.	(a – b) – c = a – (b – c)	false
8.	a × (b – c) = a × b - c	false
9.	a × (b + c) = a × b + a × c	true
10.	a ÷ (b + c) = a ÷ b + a ÷ c	false
11	(b + c) ÷ a = b ÷ a + c ÷ a	true

2.8 Approaches for dividing fractions

Refer to presentation on website
http://www.excellencegateway.org.uk/283483

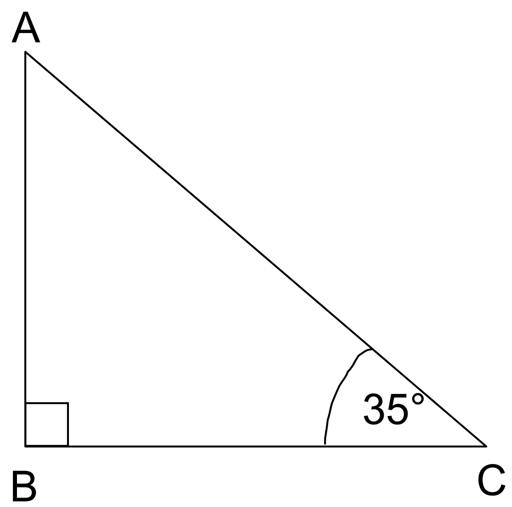
2.9 Pythagoras' theorem

Refer to presentation on website
http://www.excellencegateway.org.uk/283490

2.10 Discovering ratios in right angled triangles

Triangle A

For this exercise, you will need a protractor and ruler.
You are going to construct three similar right angled triangles with the base angle 35°.



- Use a separate sheet of plain paper for each triangle.
- For triangles 1 and 2, begin by drawing the line BC the required length.
- Construct the base angle of 35° using a protractor.
- Complete the triangle by joining B to A ensuring angle B is 90°.
- Measure AB to 1 decimal place.
- Discuss a good method for drawing triangle 3. (Hint: Angle A must be 55°).

Triangle	Length of BC	Length of AB	Ratio $^{AB}/_{BC}$ (to 2 dec places)
Triangle 1	10 cm		
Triangle 2	8 cm		
Triangle 3		10 cm	

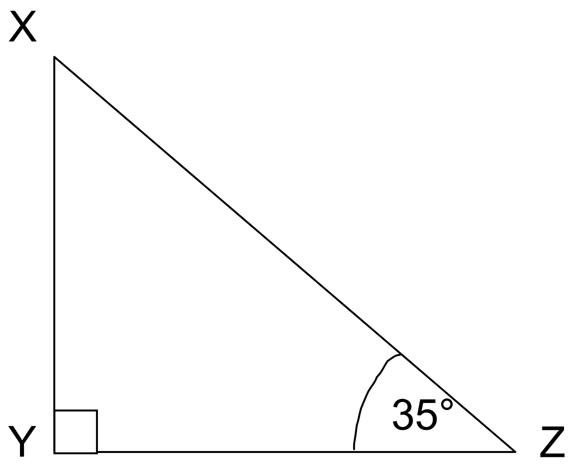
What do you notice about the ratio $^{AB}/_{BC}$?

Use your calculator to find tan 35°.

Discovering ratios in right angled triangles

Triangle B

For this exercise, you will need a protractor and ruler.
You are going to construct three similar right angled triangles with the base angle 35°.



- Use a separate sheet of plain paper for each triangle.
- For triangles 1 and 2, begin by drawing the line YZ the required length.
- Construct the base angle of 35° using a protractor.
- Complete the triangle by joining Y to X ensuring angle Y is 90°.
- Measure XZ to 1 decimal place.
- Discuss a good method for drawing triangle 3.(Hint: Angle X must be 55°).

Triangle	Length of YZ	Length of XZ	Ratio $^{YZ}/_{XZ}$ (to 2 dec places)
Triangle 1	10 cm		
Triangle 2	8 cm		
Triangle 3		10 cm	

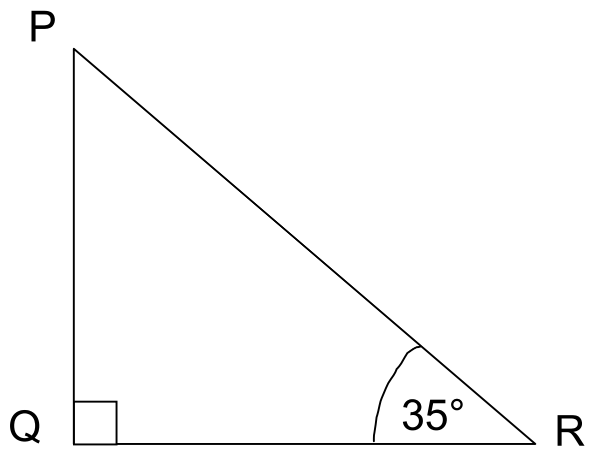
What do you notice about the ratio $^{YZ}/_{XZ}$?

Use your calculator to find cos 35°.

Discovering ratios in right angled triangles

Triangle C

For this exercise, you will need a protractor and ruler.
You are going to construct three similar right angled triangles with the base angle 35°.



Triangle	**Length of PQ**	**Length of PR**	**Ratio $^{PQ}/_{PR}$** (to 2 dec places)
Triangle 1	10 cm		
Triangle 2	8 cm		
Triangle 3		10 cm	

What do you notice about the ratio $^{PQ}/_{PR}$?

Use your calculator to find sin 35°.

Answers

Triangle	Length of BC	Length of AB	Ratio $^{AB}/_{BC}$ (to 2 dec places)
Triangle 1	10 cm	7.0 cm	0.70
Triangle 2	8 cm	5.6 cm	0.70
Triangle 3	14.5 cm	10 cm	0.70

Calculator result of **tan 35° = 0.700207538**

Triangle	Length of YZ	Length of XZ	Ratio $^{YZ}/_{XZ}$ (to 2 dec places)
Triangle 1	10 cm	12.2 cm	0.82
Triangle 2	8 cm	9.8 cm	0.82
Triangle 3	8.2 cm	10 cm	0.82

Calculator result of **cos 35° = 0.819152044**

Triangle	Length of PQ	Length of PR	Ratio $^{PQ}/_{PR}$ (to 2 dec places)
Triangle 1	10 cm	17.4 cm	0.57
Triangle 2	8 cm	14.0 cm	0.57
Triangle 3	5.7cm	10 cm	0.57

Calculator result of **sin 35° = 0.573576436**

2.11 Trigonometry

The origins of trigonometry

Two of the problems, which led to the development of trigonometry, were:

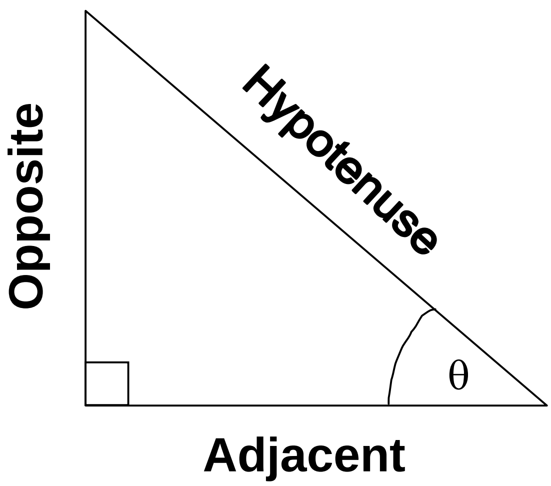
- creating a device for telling the time by using shadows which vary in length according to the time of day and height of the sun
- working out the relationship between the width of a channel in a harbour, the range of a cannon ball and the angle of the pivot of the cannon [1]

Today, trigonometry is used by engineers, surveyors, architects and others who need to work out relationships connecting distances and angles.

Right-angled triangles: the trigonometric ratios

One of the most common uses of trigonometry is in working out lengths and angles in right-angled triangles.
In calculations where an angle is involved and either you are finding the length of another side or finding an angle, trigonometry is used (Note: Pythagoras' Theorem is used when you are given two sides of a right-angled triangle and need to find the third)

Naming the sides



The **hypotenuse** is always the longest side. It is the side opposite to the right angle.

The **adjacent** side is the side next to the angle marked.

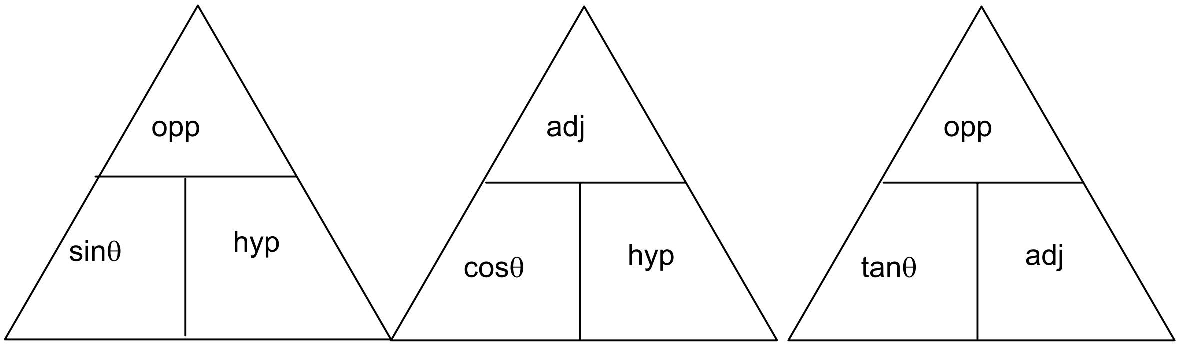
The **opposite** is the side opposite the angle marked.

The **angle** is sometimes denoted by the Greek letter theta θ.

[1] **Edexcel GCSE Mathematics Higher Course**, *Heinemann*, 2001 page 267

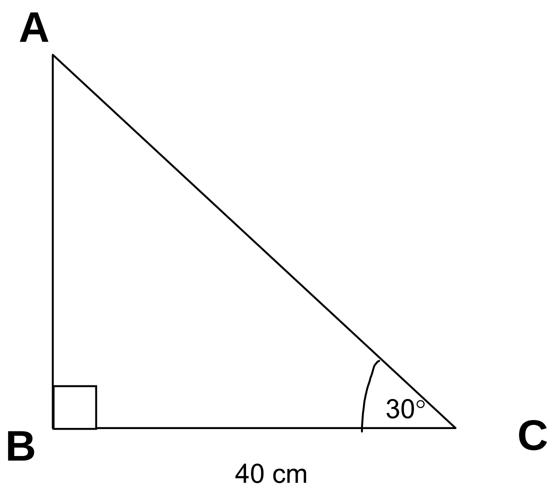
Trigonometry ratios

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} \qquad \cos\theta = \frac{\text{adj}}{\text{hyp}} \qquad \tan\theta = \frac{\text{opp}}{\text{adj}}$$



Example

In triangle ABC, the angle at the base is 30° and the length of side BC is 40 cm. Find the length of AC.



Given Adjacent side and angle
To find Hypotenuse
Trig. ratios cos ratio (look for ratios that include adj and hyp)

Trig. formula $\text{hyp} = \frac{\text{adj}}{\cos\theta}$

Calculation $\text{hyp} = \frac{40}{\cos 30^\circ}$ = 46.2 correct to 3 sig figs

The length of AC is 46.2 cm

Chapter 3
Algebra and its applications

3.1 Kinaesthetic equations

Instructions for participants

You will be given a card with an algebraic expression on it.
The teacher will give you a value for n. Substitute the value into your expression and line up in order of size with the other participants.

n^2	$\frac{1}{n + 2}$
$\frac{1}{2}(n + 1)$	$\frac{n + 1}{2}$

$4n$	$n^2 + n$
$n^2 + 1$	n^3
$\sqrt{n}$	$n - 1$

$\frac{1}{n + 1}$	$2(n + 1)$
$n^2 - 1$	n^{-1}
$n^2 - n$	$3n$

3.2 Algebra carousel activity

Plan

45 mins	**Carousel of advanced numeracy activities** See list of activities and instructions below. One activity to be set out per table, tables in cabaret style around the room (if short of space, activities can be placed face-down in the centre of each table until required). Participants move from one table to another to try out the activities, and use review sheet to record comments. If the group is large, they should be split into two. While one group carry out the carousel, the other group work through paper based packs on fundamental algebra skills. Trainer to intervene and provide support with activities where requested. Trainer should emphasise that participants are not expected to be able to do all the activities – the idea is for them to assess which ones they are confident on, which they need to brush up on and which they need a lot of help with – and the Algebra Self Assessment Record Sheet is to record this.	Algebra activity sheets (a – d) Carousel Review sheet

List of Carousel Activities

Expressions and equations: true /false (Activity a)
Paper exercise – Photocopy a pile of the activity and leave on the table. Laminate answer sheet

Matching linear and quadratic graphs (Activity b)
Matching activity – Colour photocopy, laminate and cut out cards. Laminate answer sheet

Real life graphs (Activity c)
Matching activity – Colour photocopy, laminate and cut out cards. Laminate answer sheet

Formulae matching (Activity d)
Matching activity – Enlarge to A3, colour photocopy, laminate and cut out cards. Laminate answer sheet

Linking algebra with number (Activity e)
Group discussion – Photocopy a pile of the activity and leave on the table. Laminate possible solution sheet. Provide some counters or blocks.

Algebra carousel Activity A (1 copy per participant)

Expressions and equations: True/false activity

$x + 5$ and $x^2 + 5$

Complete this grid with your answers

Which of the statements are:
definitely true, definitely false or sometimes true?

Statement	$x + 5$	$x^2 + 5$
Equation		
Expression		
Equals (-4)		
Quadratic		
Cubic		
Equals zero		
Equals 105		
A term		
Linear		

Expressions and equations: True/false activity

Answers

Statement	$x + 5$	$x^2 + 5$
Equation	Definitely false	Definitely false
Expression	Definitely true	Definitely true
Equals (-4)	Possibly true	Possibly true*
Quadratic	Definitely false	Definitely true
Cubic	Definitely false	Definitely false
Equals zero	Possibly true	Possibly true*
Equals 105	Possibly true	Possibly true
A term	Definitely false	Definitely false
Linear	Definitely true	Definitely false

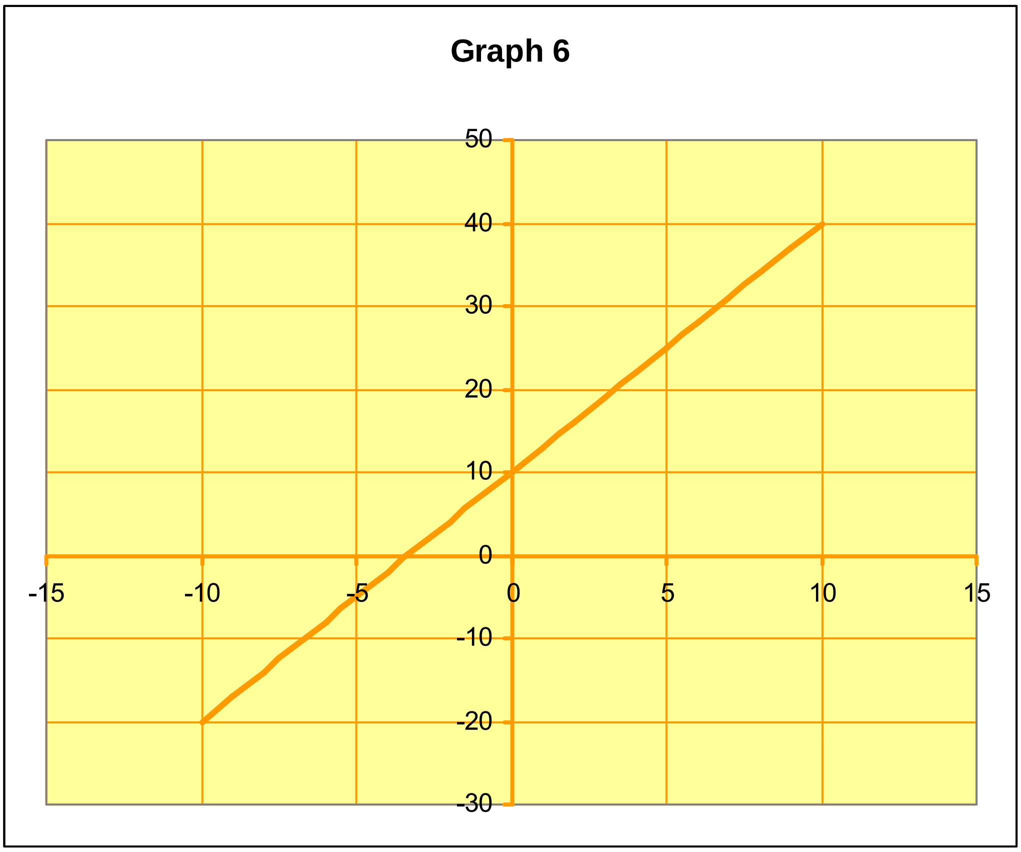
* Please note that it is possible to have the square root of a negative number in the branch of mathematics called **complex numbers**.

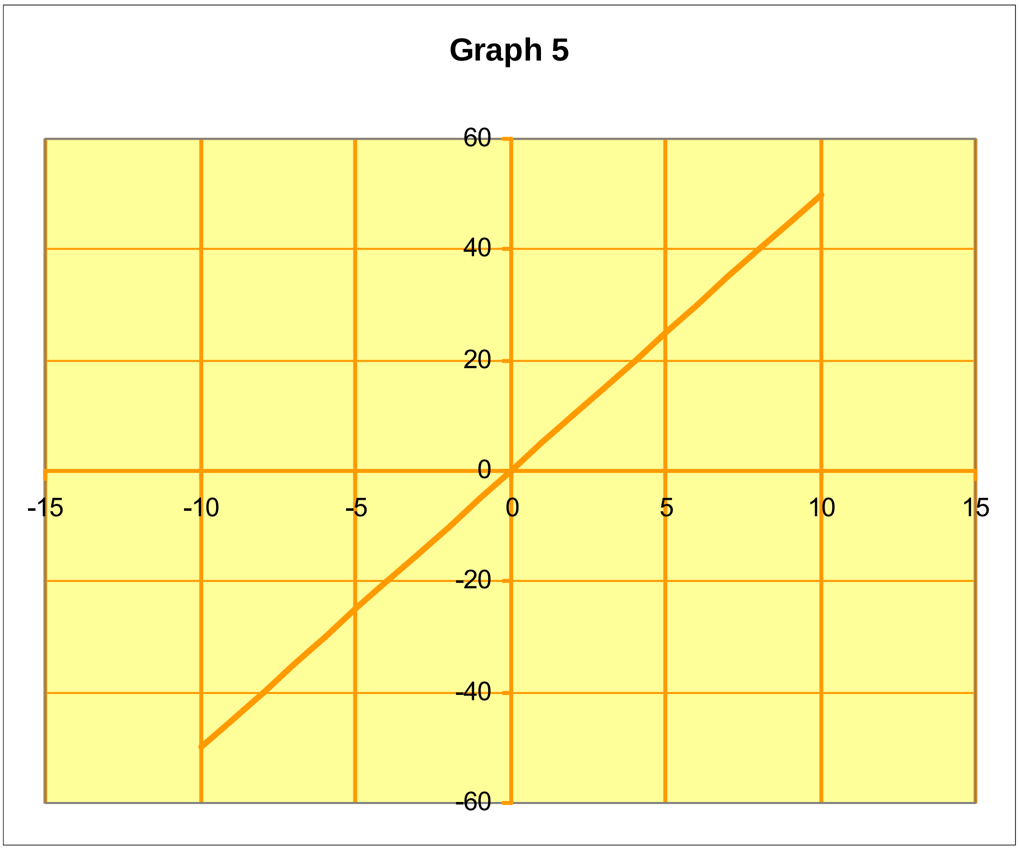
Algebra carousel Activity B (to be laminated and cut into cards)

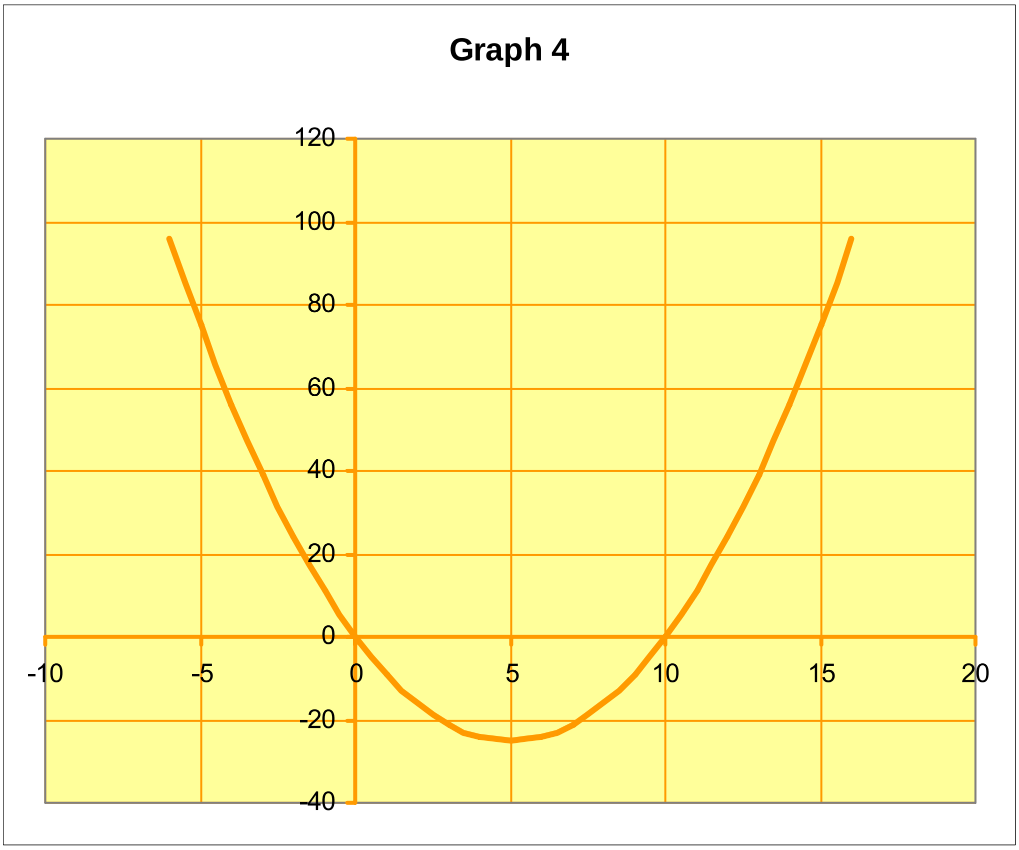
Matching linear and quadratic graphs

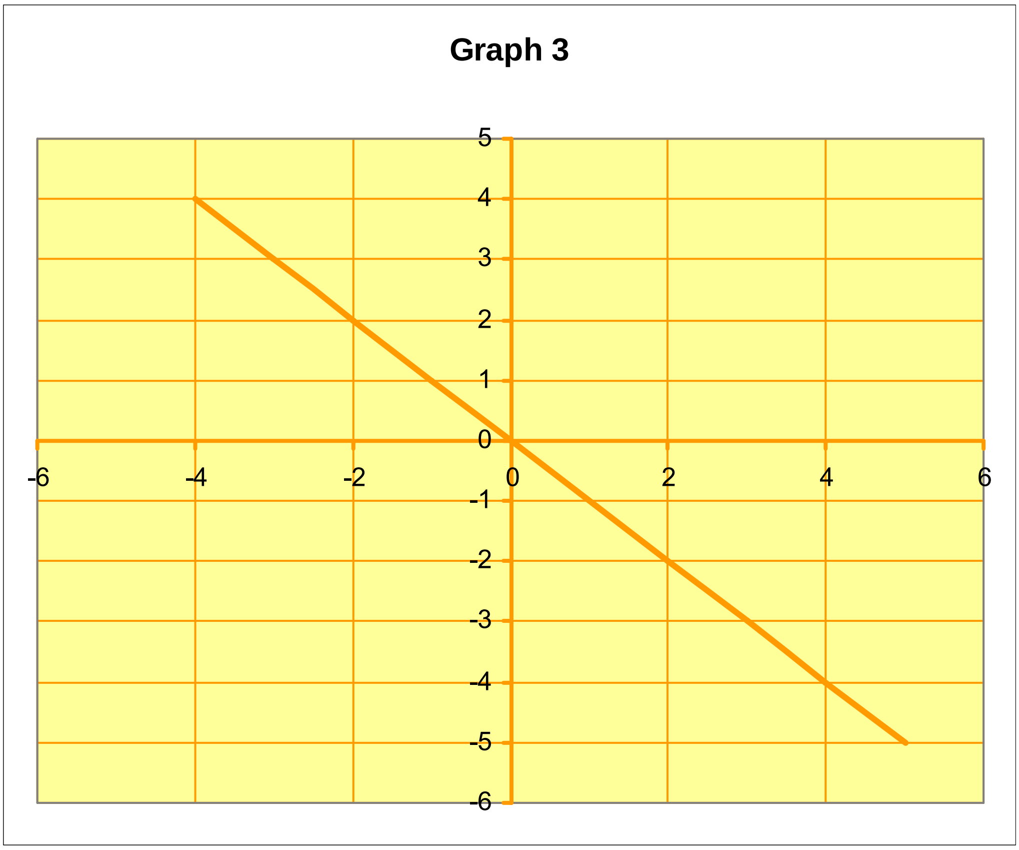
Instructions

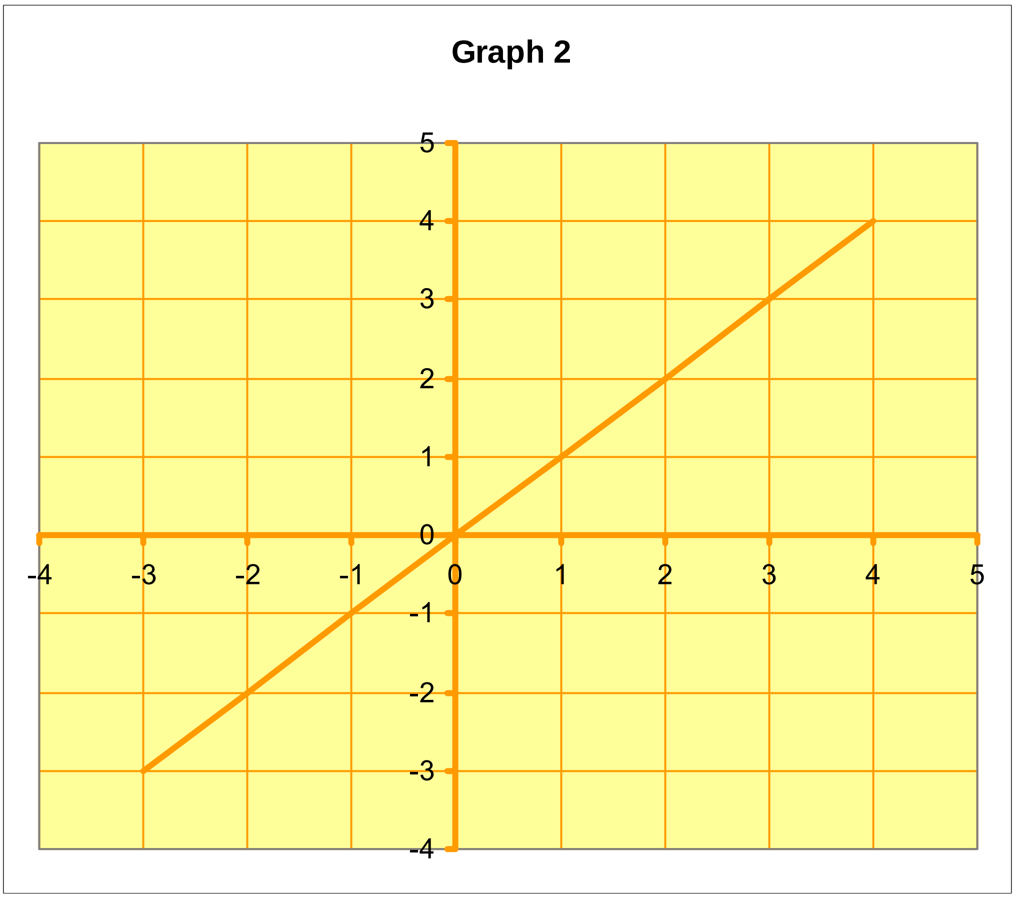
- Match the graphs to the equations
- Discuss the difference between the equation of a straight line graph and a curved graph (parabola)

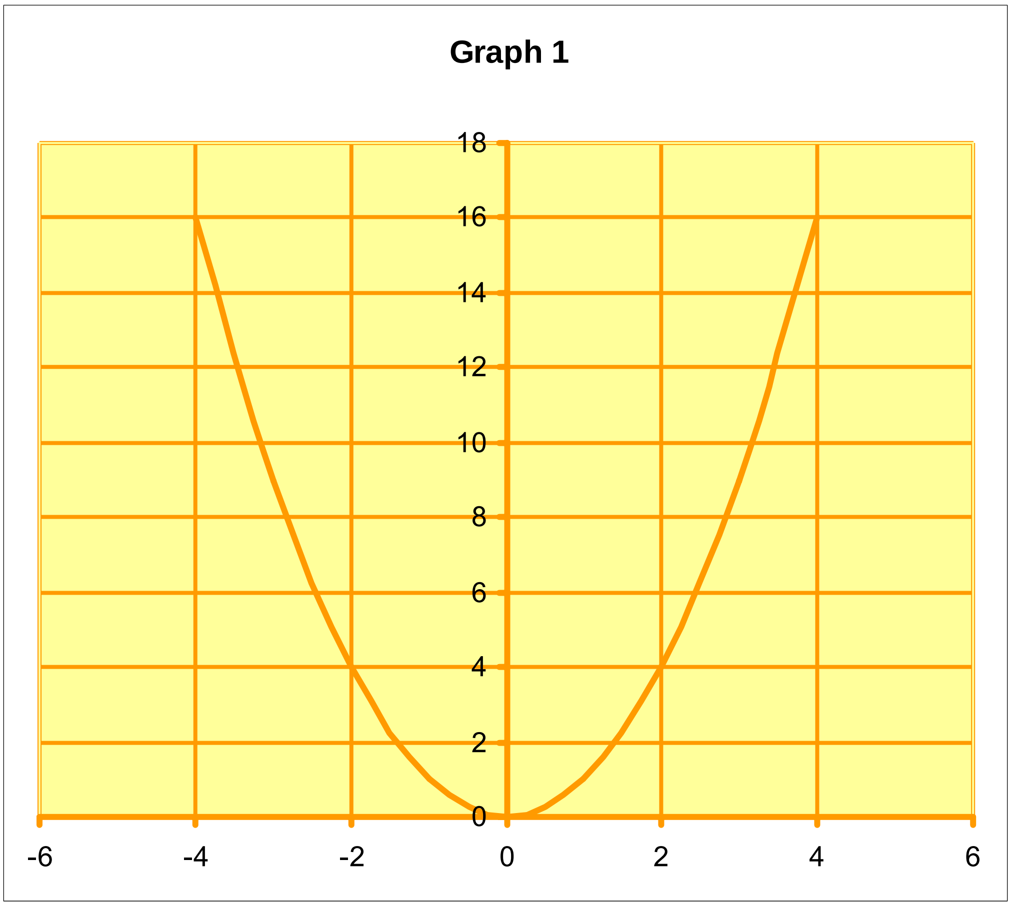



Graph 5
60
40
20
0
-20
-40
-60
-15
-10
-5
0
5
10
15


Graph 4
120
100
80
60
40
20
0
-20
-40
-10
-5
0
5
10
15
20


Graph 3
5
4
3
2
1
0
-1
-2
-3
-4
-5
-6
-6
-4
-2
0
2
4
6


Graph 2
5
4
3
2
1
0
-1
-2
-3
-4
-4
-3
-2
-1
0
1
2
3
4
5



Graph 1
18
16
14
12
10
8
6
4
2
0
-6
-4
-2
0
2
4
6

$$y = 3x + 10$$

$$y = 5x$$

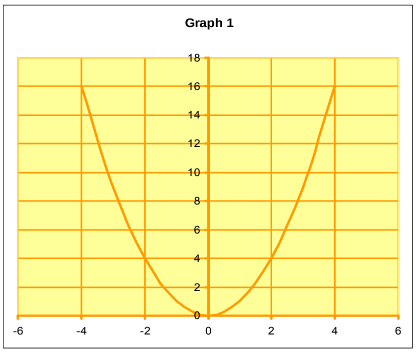
$$y = x^2 - 10x$$

$$y = -x$$

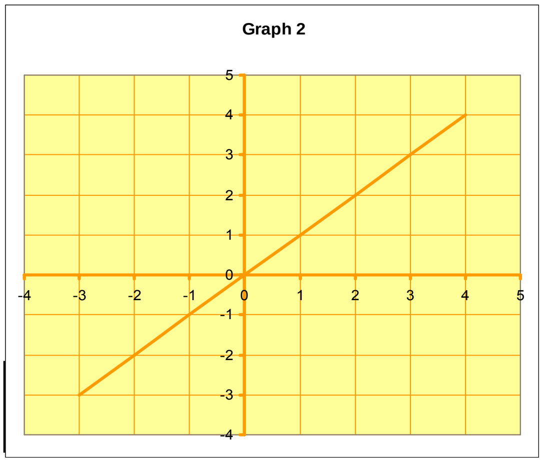
$$y = x^2$$

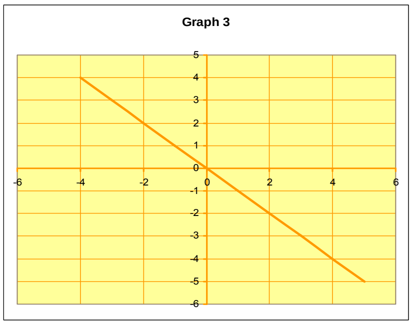
Matching linear and quadratic graphs

Answers

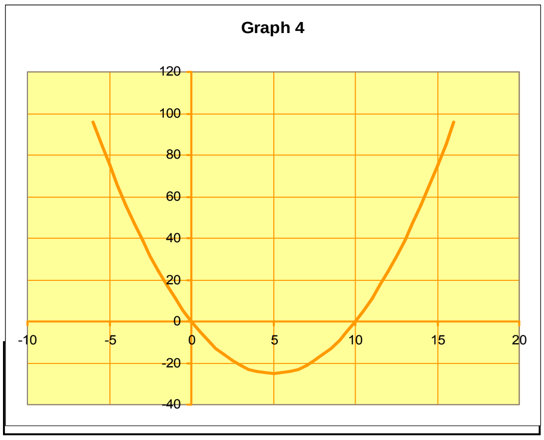


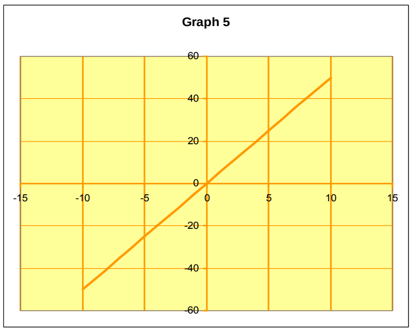
$y = x^2$



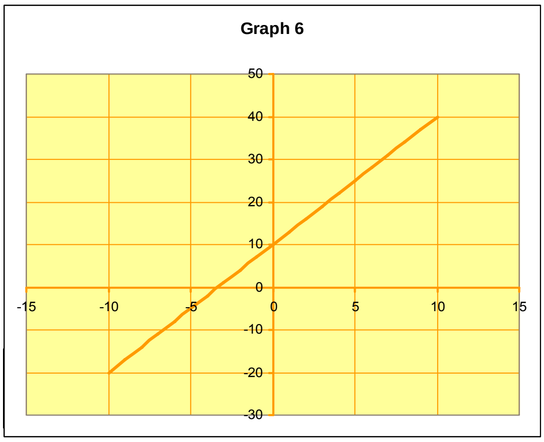


$y = -x$





$y = 5x$



Algebra carousel Activity C (to be laminated and cut into cards)

Real life graphs

Instructions
You are given three scenarios and three straight line graphs.   • Match the scenarios with the graph (lilac)   • Match the equations with the graphs (yellow).   • Answer the extension question (green).

How much water had been let out of the bath after 2½ minutes?

I need 100 cards for my wedding. How much will it cost?

How much is £15.00 in Euros?

This graph shows the cost of printing wedding invitation cards. It gives the cost against the number of cards.

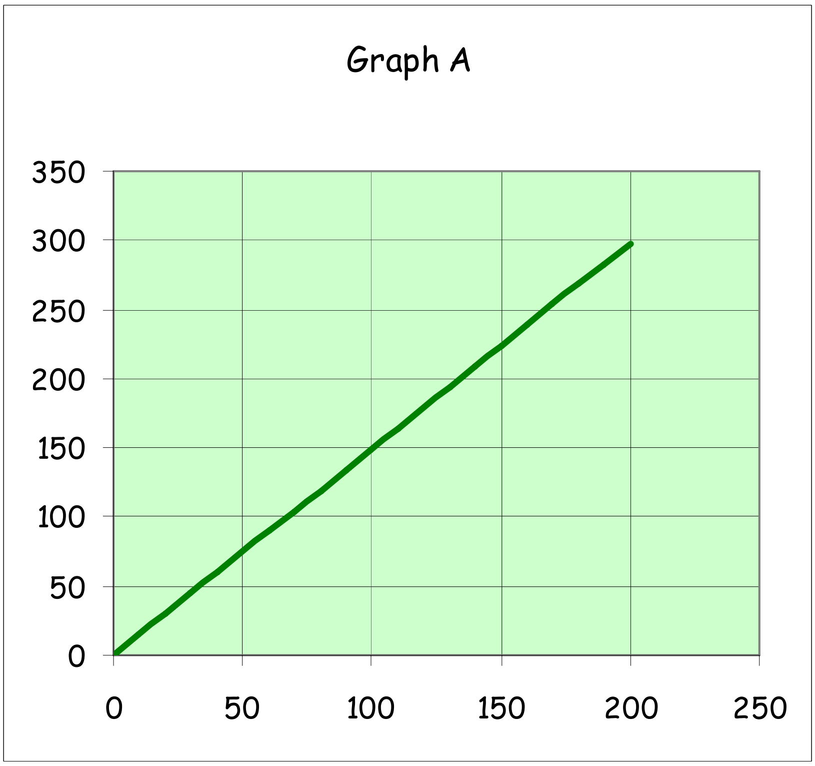
This graph shows water being let out of a bath. It gives the capacity in litres against the time.

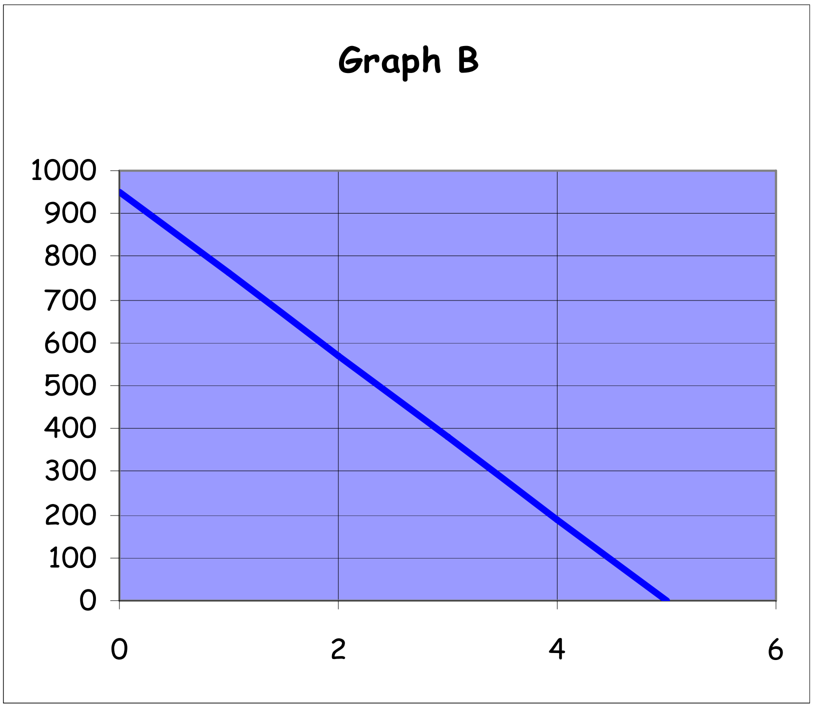
This graph shows the conversion of Euros against pounds.

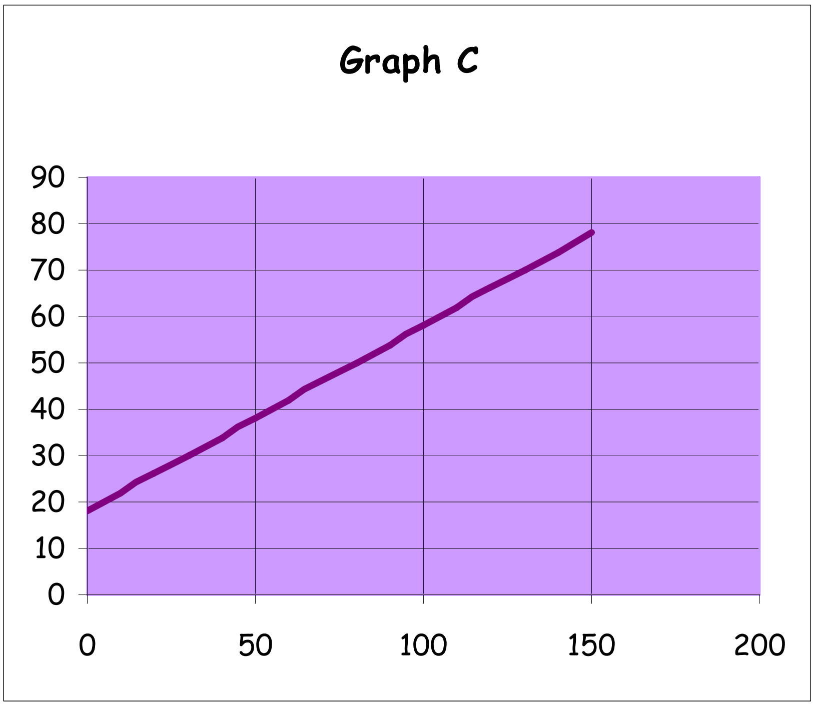
$$y = 1.49x$$

$$y = 950 - 190x$$

$$y = 0.4x + 18$$

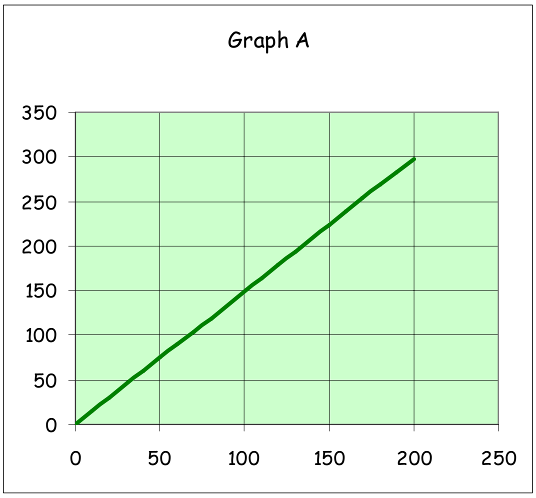

Graph A
350
300
250
200
150
100
50
0
0
50
100
150
200
250


Graph B
1000
900
800
700
600
500
400
300
200
100
0
0
2
4
6


Graph C
90
80
70
60
50
40
30
20
10
0
0
50
100
150
200

Real life graphs

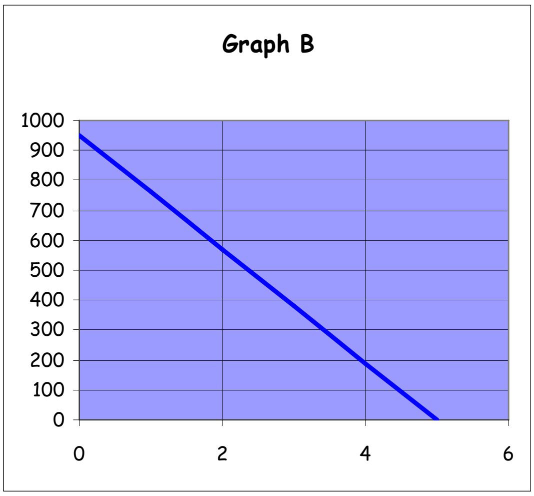
Answers



This graph shows the conversion of Euros against pounds.

y = 1.49x

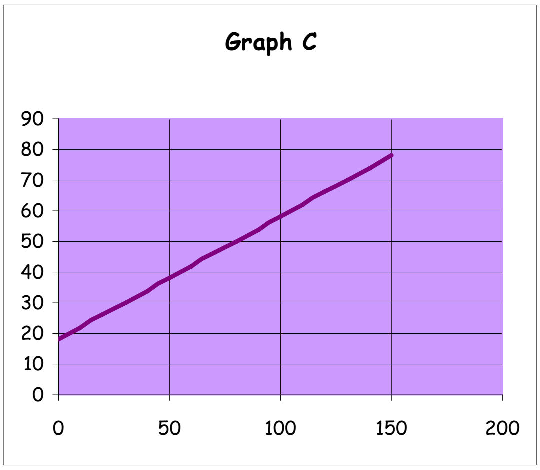
How much is £15.00 in Euros?
Answer: *€22.35*



This graph shows water being let out of a bath. It gives the capacity in litres against the time.

y = 950 - 190x

How much water had been let out the bath after 2½ minutes?
Answer: *475 litres*



This graph shows the cost of printing wedding invitation cards. It gives the cost against the number of cards.

y = 0.4x + 18

I need 100 cards for my wedding. How much will it cost?
Answer: *£58.00*

Algebra carousel Activity d (to be laminated and cut into cards)

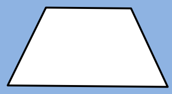
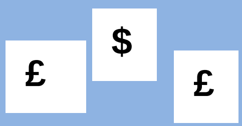
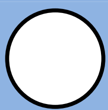
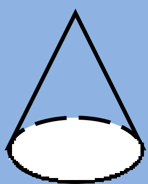
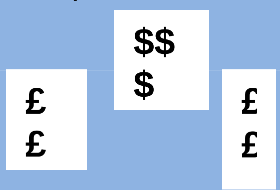
Formula matching

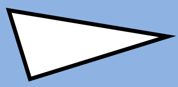
Instructions

You are given three sets of cards

- Match the name with the descriptions of the formula in words with the formula in symbols

- There are six cards which have no match

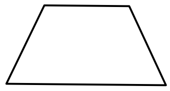
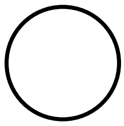
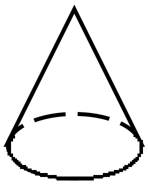
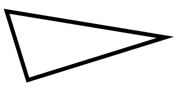
Area of Trapezium 	Add the lengths of the two parallel sides and halve the answer. Multiply this by the perpendicular height.	$A = \frac{1}{2}(a + b)h$
Simple Interest Earned 	Multiply together the amount of money invested, the length of time for which it is invested and the percentage rate of interest. Divide the answer by 100.	$I = \frac{PRT}{100}$
Area of a circle 	Take the measure of the radius and multiply it by itself. Multiply this result by the constant *pi.*	$A=\pi r^2$
Volume of a cone 	Take the measure of the radius of the base and multiply this by itself. Multiply this by the perpendicular height of the cone and then by the constant *pi.* Divide this answer by three.	$V=\frac{1}{3}\pi r^2 h$
Compound Interest 	Divide the percentage interest rate by 100 and add one. “Raise” the answer to the power given by the number of time periods (e.g. years, months) for which the amount is invested. Multiply by the amount originally invested.	$A = P(1 + \frac{r}{100})^n$

Pythagoras' Theorem 	Square the lengths of the two shorter sides. Add these answers. Take the square root of this answer	$c = \sqrt{a^2 + b^2}$

$A = \frac{1}{2}a + bh$	$I = \frac{P + R + T}{100}$	$A = 2\pi r$
$V = \frac{1}{3}\pi(rh)^2$	$C^2 = \sqrt{a + b}$	$A = P + 1(\frac{r}{100})^n$

Formula matching

Answers

Area of Trapezium 	Add the lengths of the two parallel sides and halve the answer. Multiply this by the perpendicular height.	$A = \frac{1}{2}(a + b)h$
Simple Interest Earned $ £ £	Multiply together the amount of money invested, the length of time for which it is invested and the percentage rate of interest. Divide the answer by 100.	$I = \frac{PRT}{100}$
Area of a circle 	Take the measure of the radius and multiply it by itself. Multiply this result by the constant *pi.*	$A = \pi r^2$
Volume of a cone 	Take the measure of the radius of the base and multiply this by itself. Multiply this by the perpendicular height of the cone and then by the constant *pi.* Divide this answer by three.	$V = \frac{1}{3}\pi r^2 h$
Compound Interest $$ $ £ £ £ £	Divide the percentage interest rate by 100 and add one. "Raise" the answer to the power given by the number of time periods (eg years, months) for which the amount is invested. Multiply by the amount originally invested.	$A = P(1 + \frac{r}{100})^n$
Pythagoras' Theorem 	Square the lengths of the two shorter sides. Add these answers. Take the square root of this answer	$c = \sqrt{a^2 + b^2}$

These have no match

$A = \frac{1}{2}a + bh$	$I = \frac{P + R + T}{100}$	$A = 2\pi r$
$V = \frac{1}{3}\pi(rh)^2$	$C^2 = \sqrt{a + b}$	$A = P + 1(\frac{r}{100})^n$

Algebra carousel Activity e (photocopy)

Linking algebra with number

A number squared is always one more than the product of the numbers either side of the number.

For example:

$5^2 = (6 \times 4) + 1$

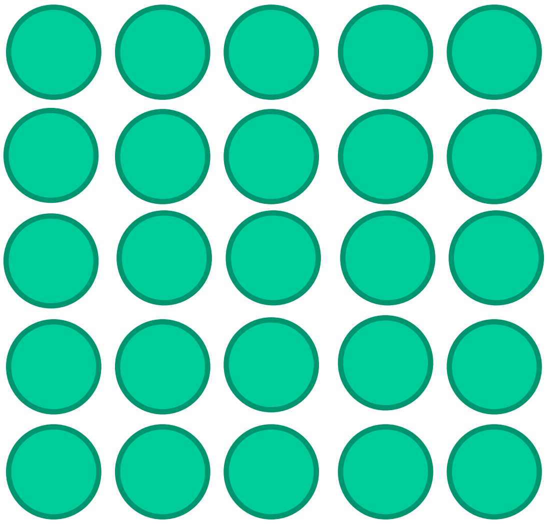
Prove/argue

Either by using algebra or shape and space

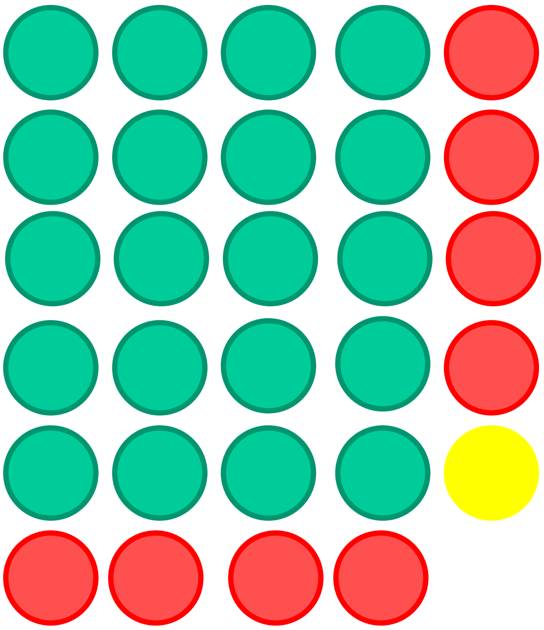
Linking algebra with number

Possible solutions

Using shapes
5^2

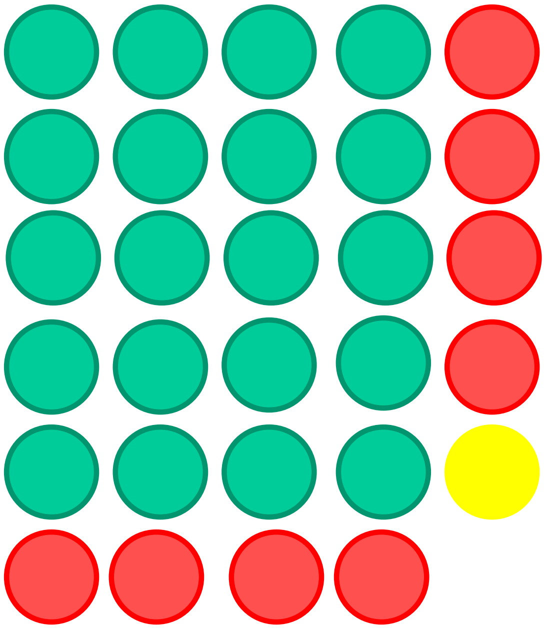


(6 x 4) + 1 = 25



Using algebra

$(x - 1)(x + 1) = x^2 - x + x - 1 = x^2 - 1$



Algebra self assessment record sheet

Activity	*How I did on this*	*What else I need to know*
Expressions and equations: true/false		
Matching linear and quadratic graphs		
Real life graphs		
Formula matching		
Linking algebra with number		

3.3 I think of a number

I think of a number	n
I double it	2n
I add 10	2n + 10
I get 50	2n + 10 = 50
What is my number?	n = 20

I think of a number	n
I square it	n^2
I add 5	$n^2 + 5$
I get 30	$n^2 + 5 = 30$
What is my number?	n = 5 or n = -5*

*n can be a negative number

Instructions for participants

Work in pairs, but individually write down an equation. Do not show this to your partner. Take turns to talk through this equation with your partner using the type of language shown in the example e.g. I think of a number, I double it etc. Your partner will try to record these steps in the tables below as you do this. Check the resulting equation with your equation – they should be the same. Discuss any differences between what you have and what your partner has written and why you think they have occurred. Try to solve the equations and compare your methods.

I think of a number	n

I think of a number	n

Instructions for participants

Work in pairs, but individually write down an equation. Do not show this to your partner. Take turns to talk through this equation with your partner using the type of language shown in the example e.g. I think of a number, I double it etc. Your partner will try to record these steps in the tables below as you do this. Check the resulting equation with your equation – they should be the same. Discuss any differences between what you have and what your partner has written and why you think they have occurred. Try to solve the equations and compare your methods.

I think of a number	n

I think of a number	n

3.4 Matching expressions

Instructions for participants
Match the word problems to the expression cards.
Check your answers and discuss any issues.

A boy is **x** years old now. How old was he 5 years ago?

Find the total cost of 3 pencils at **x** pence each and 8 pens at **y** pence each.

A person works **x** hours per weekday, on Saturday **y** hours and on Sunday **z** hours; how many hours does he work per week?

What is the perimeter of a rectangle **x** mm long and **y** mm wide?

How many minutes are there between **x** minutes to 10 o'clock and 12 o'clock?

x articles are bought for a total of **y** pence. Find the cost in pounds of buying **z** articles at the same rate.

A householder buys two daily papers (Monday to Saturday) at **x** pence each and 3 Sunday papers at **y** pence each. What is the yearly expenditure (in pounds) on newspapers?

During a sale a shop gives a reduction of **n** pence in the pound on the marked price of articles. If a customer buys articles marked at £**x,** £**y** and £**z** how much will she actually pay?

After spending one-seventh of my income on rent and two-sevenths of the remainder on household expenses I have £**x** left. What is my income?

The cost of a supply of electricity is as follows. There is a fixed charge of £**x,** for the rent of the meter the charge is £**y** and the electricity is charged for at **z** pence per unit. If **n** units of electricity are used, find an expression for the total cost (in £).

(x - 5) years	£ $\frac{zy}{100x}$
(3x + 8y) pence	£ $\frac{39(4x + y)}{25}$
(5x + y + z) hours	£ $(x + y + z)(1 - \frac{n}{100})$
2(x + y) mm	£ $\frac{49x}{30}$
(120 + x) minutes	£$(x + y + \frac{zn}{100})$

Answers for expressions

A boy is x years old now. How old was he 5 years ago? $(x - 5)$ years
Find the total cost of 3 pencils at x pence each and 8 pens at y pence each. $(3x + 8y)$ pence
A person works x hours per weekday, on Saturday y hours and on Sunday z hours; how many hours does he work per week? $(5x + y + z)$ hours
What is the perimeter of a rectangle x mm long and y mm wide? $2(x + y)$ mm
How many minutes are there between x minutes to 10 o'clock and 12 o'clock. $(120 + x)$ minutes
x articles are bought for y pence. Find the cost in pounds of buying z articles at the same rate. £ $\frac{zy}{100x}$
A householder buys two daily papers at x pence each and 3 Sunday papers at y pence each. What is the yearly expenditure (in pounds) on newspapers? £ $\frac{39(4x + y)}{25}$
During a sale a shop gives a reduction of n pence in the pound on the marked price of articles. If a customer buys articles marked at £x, £y, and £z how much will she actually pay? £ $(x + y + z)(1 - \frac{n}{100})$
After spending one-seventh of my income on rent and two-sevenths of the remainder on household expenses I have £x left. What is my income? £ $\frac{49x}{30}$
The cost of a supply of electricity is as follows. There is a fixed charge of £x, for the rent of the meter the charge is £y and the electricity is charged for at z pence per unit. If n units of electricity are used find an expression for the total cost (in £). £$(x + y + \frac{zn}{100})$

Solutions:

A boy is **x** years old now. How old was he 5 years ago?

Subtract 5 years from his current age to work this out:
x – 5 years

Find the total cost of 3 pencils at **x** pence each and 8 pens at **y** pence each.

*Each pencil costs **x** pence*
*Cost of 3 pencils = 3**x** pence*
*Each pen costs **y** pence*
Cost of 8 pens = 8y pence
Total cost = 3x + 8y pence

A person works **x** hours per weekday, on Saturday **y** hours and on Sunday **z** hours; how many hours does he work per week?

*There are 5 weekdays, so total hours worked on weekdays = 5**x***
Total hours worked per week = 5x + y + z hours

What is the perimeter of a rectangle **x** mm long and **y** mm wide?

Perimeter of a rectangle = 2 x length + 2 x width
Perimeter of this rectangle = 2x + 2y mm

How many minutes are there between **x** minutes to 10 o'clock and 12 o'clock?

*There are **x** minutes before 10 o'clock and 120 minutes between 10 o'clock and 12 o'clock.*
Total minutes = x + 120 minutes

x articles are bought for a total of **y** pence. Find the cost in pounds of buying **z** articles at the same rate.

Rate or unit cost = $\frac{y}{x}$ *pence*

Multiply by z to get cost in pence of buying z articles = $\frac{zy}{x}$

Divide by 100 to convert cost to £: $\pounds\,\frac{zy}{100x}$

A householder buys two daily papers (Monday to Saturday) at **x** pence each and 3 Sunday papers at **y** pence each. What is the yearly expenditure (in pounds) on newspapers?

Cost of 12 daily papers (two daily papers Monday to Saturday) = 12x pence
Cost of 3 Sunday papers = 3y pence
Weekly cost = 12x + 3y pence
Annual cost in pence = 52(12x + 3y)pence
Annual cost in £ = $\pounds\,\frac{52(12x + 3y)}{100}$
This simplifies to: $\pounds\,\frac{13\,(12x + 3y)}{25}$
Taking the factor of 3 out of the bracket, this further simplifies to:

$$\pounds\,\frac{39(4x + y)}{25}$$

During a sale a shop gives a reduction of **n** pence in the pound on the marked price of articles. If a customer buys articles marked at £**x,** £**y,** and £**z** how much will she actually pay?

Before the sale, total cost = £(x + y + z)
Total reduction in pence = n(x + y + z)
Total reduction in £ = $\pounds\,\frac{n(x + y + z)}{100}$

During the sale, customer pays: $£(x + y + z) - \frac{n(x + y + z)}{100}$

Factorising, this simplifies to:

$$£\,(x + y + z)\left(1 - \frac{n}{100}\right)$$

After spending one-seventh of my income on rent and two-sevenths of the remainder on household expenses I have £**x** left. What is my income?

After spending one-seventh of my income on rent I will have $^{6}/_{7}\ I$ *remaining (where* I *= income)*
I spend two-sevenths of the remainder:
Two-sevenths of the remainder = $^{2}/_{7} \times {}^{6}/_{7}\ I$

$= {}^{12}/_{49}\ I$
This leaves me with: $^{6}/_{7}\ I - {}^{12}/_{49}\ I = £\mathbf{x}$
Using a common denominator of 49:
$^{42}/_{49}\ I - {}^{12}/_{49}\ I = £x$
$^{30}/_{49}\ I = £x$
Multiply both sides by 49:
$30\ I = £49x$
Divide both sides by 30:

$$\mathbf{I = £\,\frac{49x}{30}}$$

The cost of a supply of electricity is as follows. There is a fixed charge of £ **x,** for the rent of the meter the charge is £**y** and the electricity is charged for at **z** pence per unit. If **n** units of electricity are used, find an expression for the total cost (in £).

Fixed charge plus rent of meter = £x + £y
Cost of units of electricity = zn pence
Cost of units of electricity in £ = $£\frac{zn}{100}$

$$\textbf{Total cost} = £\left(x + y + \frac{zn}{100}\right)$$

3.5 Simultaneous equations – ordering steps

Instructions

Order the steps shown on the blue cards for solving the equations:

$$3x + 2y = 12 \text{ and } 5x - 2y = 4$$

Match the blue cards with the green cards to show the results of carrying out each step

Label equations (1) and (2)	$3x + 2y = 12$ (1) $5x - 2y = 4$ (2)
Check to see if there are the same number of xs or ys in both equations	There are the same number of ys
Decide whether to add or subtract equations to eliminate x or y	Add equations as the y signs are different
Add the equations together	$3x + 2y = 12$ (1) $\underline{5x - 2y = 4}$ (2) $8x = 16$
Solve the equation to find the value of one of the letters	$x = 2$
Substitute the value you have found into one of the original equations	$3x + 2y = 12$ (1) $6 + 2y = 12$
Solve the equation to find the value of the remaining letter	$6 + 2y = 12$ $2y = 6$ $y = 3$
Check solutions work by substituting both values into the other original equation	$x = 2$; $y = 3$ $5x - 2y = 4$ (2) $10 - 6 = 4$

Simultaneous equations – ordering steps

Answers

Label equations (1) and (2)	3x + 2y = 12 (1) 5x - 2y = 4 (2)
Check to see if there are the same number of xs or ys in both equations	There are the same number of ys
Decide whether to add or subtract equations to eliminate x or y	Add equations as the y signs are different
Add the equations together	3x + 2y = 12 (1) 5x - 2y = 4 (2) 8x = 16
Solve the equation to find the value of one of the letters	x= 2
Substitute the value you have found into one of the original equations	3x + 2y = 12 (1) 6 + 2y = 12
Solve the equation to find the value of the remaining letter	6 + 2y = 12 2y = 6 y = 3
Check solutions work by substituting both values into the other original equation	x = 2; y = 3 5x - 2y = 4 (2) 10 – 6 = 4

3.6 Writing equations and solving

Instructions for participants

Write the pair of simultaneous equations for each question and then solve them. Check your answers against the answer sheet.

1) Divide the number 12 up into two parts so that 5 times the first part added to 3 times the second part equals 40.

2) Two students A and B have £120 in cash between them. If A gives B £10, B would have twice as much as A. How much has A?

3) 15 articles are bought. Some cost 5 pence each and the others cost 8 pence each. If the total amount paid for them is 90 pence how many of each is bought?

4) Find three consecutive whole numbers so that their sum is 48.

5) Lift A can carry 4 more people than lift B. With both lifts full, when B makes three journeys it carries as many people as A does in two journeys. Find how many people each of the lifts can carry.

6) In a club share-out £1720 is to be shared between 200 members. Full members are to receive £10 each, associate members £8 each and provisional members £4 each. If there are 5 times as many full members as provisional members, how many associate members are there?

Answers for equations

1) x + y =12 5x + 3y = 40 **x = 2 and y = 10**
2) A + B = 120 B + 10 = 2(A – 10) A = 50 A has £50
3) x + y =15 5x + 8y = 90 x = 10 and y = 5 10 articles at 5p and 5 articles at 8p
4) x + (x + 1) + (x +2) = 48 **x = 15** **The numbers are 15, 16 and 17**
5) A = B + 4 2A = 3B A = 12; B = 8 A carries 12 people; B carries 8 people
6) a + f + p = 200 8a + 10f + 4p = 1720 f = 5p a = 80 There are 80 associate members

Chapter 4
Probability and statistics

4.1 Probability bingo

Instructions for participants

You have been given a bingo card with some fractions on and a highlighter.
The teacher will ask you some questions on probability. Work out the answer to each question and if the answer corresponds to one of the fractions on your card, cross it through with the highlighter.
If you get to a point where you have crossed through all of the fractions on your card, call out 'bingo'.

$^1/_6$		$^5/_{26}$	$^1/_{13}$		$^1/_{12}$
$^5/_{12}$		$^1/_4$		$^1/_7$	$^2/_{11}$

	$^1/_2$		$^1/_{13}$	$^1/_7$	$^1/_{12}$
$^5/_{12}$	$^1/_{52}$		0		$^5/_{26}$

$^1/_6$		$^5/_{26}$		$^1/_3$	$^1/_{12}$
$^5/_{12}$	$^1/_{52}$	$^1/_4$	0		

	$^1/_2$	$^2/_{11}$		$^1/_3$	
$^5/_{12}$		$^1/_4$	0	$^1/_7$	$^1/_{52}$

$^1/_6$	$^1/_2$	$^2/_{11}$		$^1/_3$	
	$^1/_{12}$		0	$^1/_7$	$^1/_{52}$

$^1/_6$	$^1/_2$		$^1/_{13}$	$^1/_3$	
	$^1/_{52}$	$^1/_4$		$^1/_7$	$^5/_{26}$

0	$^1/_6$		$^1/_3$	$^1/_4$	
	$^1/_{12}$	$^2/_{11}$		$^1/_{13}$	$^5/_{12}$

The probability of getting a 6 on a roll of a fair six sided die	$^1/_6$
The probability of getting a head on a toss of a fair coin	$^1/_2$
The probability of getting a king from a pack of 52 playing cards	$^1/_{13}$
The probability of getting a square number when a fair six sided die is rolled	$^1/_3$
The probability of choosing a day of the week beginning with the letter M	$^1/_7$
The probability of choosing a vowel from the letters in the alphabet	$^5/_{26}$
The probability of getting the ace of hearts from a pack of 52 playing cards	$^1/_{52}$
The probability of choosing an M from the word MATHEMATICS	$^2/_{11}$
The probability of choosing a month beginning with the letter S	$^1/_{12}$
The probability of choosing a prime number from a dodecahedron dice	$^5/_{12}$
The probability of getting 10 with a six sided die	0
The probability of getting a club from a pack of 52 playing cards	$^1/_4$

4.2 Probability scale

In your group, look at the probability event cards you have been given and discuss the likelihood of those events happening. Then decide between you where you would place them on a probability scale. Once you have come to an agreement, place the event cards on the probability scale you have been given.
Once you have done this, think of up to three more events and write them on the blank cards. Discuss the likelihood of these events happening and place them on the probability scale.

When your group has placed all of the cards you have been given, have a look at how other groups have placed their event cards on their probability scale. Do you agree with their placements? Do they differ from how your group has placed the cards?

Refer back to the large probability scale which has the probability terms placed on it. Does the placement of the probability terms tie in with where you have placed your event cards or would you like to make any changes?

The probability of there being no news on TV tomorrow

The probability of Labour winning the next General Election

possibly

maybe

sometimes

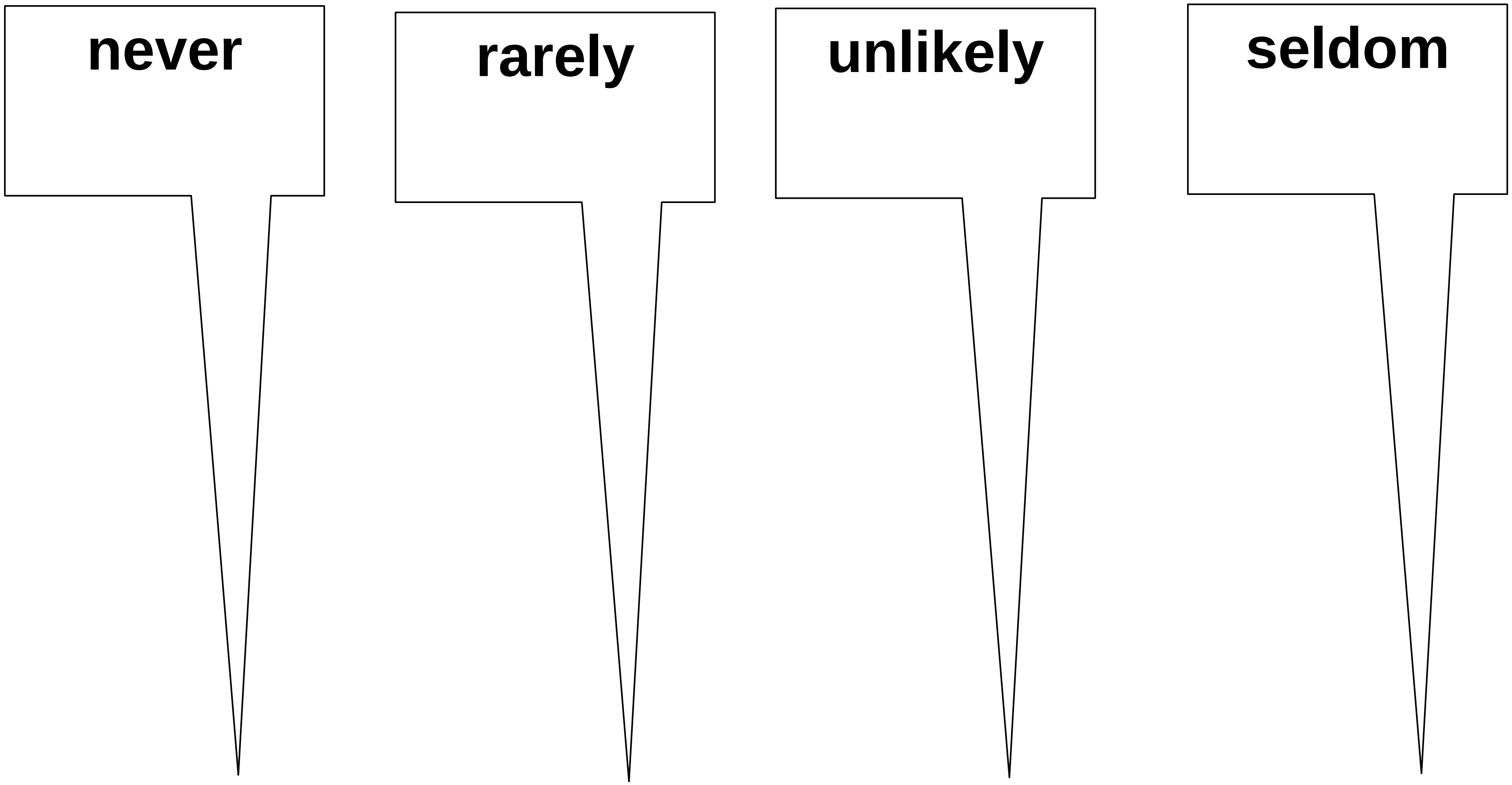
no chance

frequently

often

always

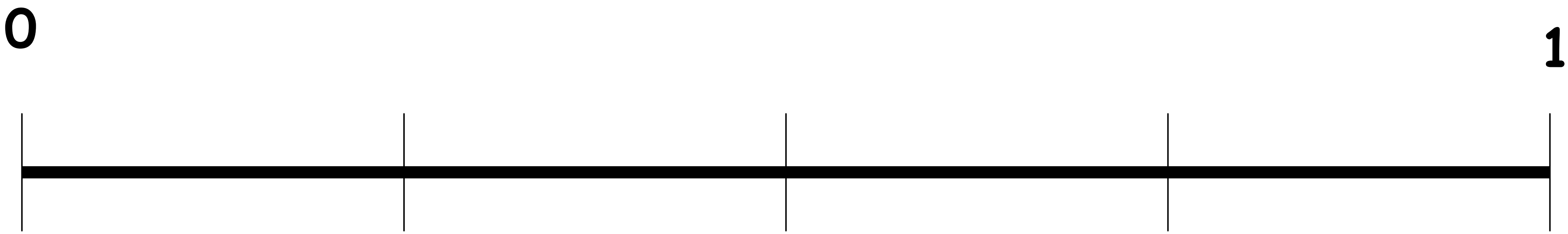
certain

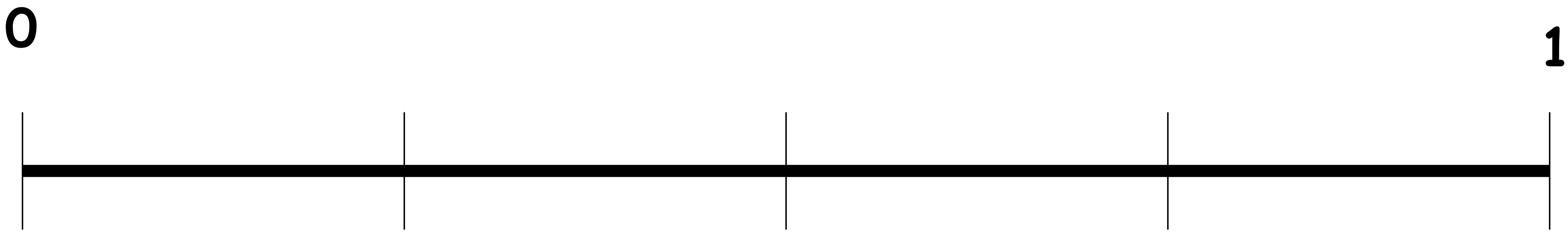

never
rarely
unlikely
seldom

likely

quite likely

very likely


0
1


0
1

4.3 Independent vs dependent events

Laminate and cut into cards

Tossing a head on a 10p coin and tossing a head on a £1 coin	Choosing a girl for my football team, then choosing another girl
Rolling a six on one die and rolling a five on another die	Picking an ace from a pack, keeping it and picking another ace
Choosing a red ball from a bag, replacing it, then a black ball	Choosing a yellow sweet from a bag, eating it and choosing a red sweet
Choosing a king from a pack of cards, replacing it and choosing another king	**Instructions** Consider the following questions. Decide whether they are dependent or independent events and sort the cards accordingly.

Extension (optional): calculate the probabilities.

1. What is the probability of tossing a head on a 10p coin and tossing a head on a £1 coin?

2. What is the probability of rolling a *six* on one die and rolling a *five* on another die?

3. There are 6 boys and 6 girls. I am going to choose five people for my football team. What is the probability of choosing a girl, then another girl?

4. A bag contains 4 red sweets and 7 yellow sweets. I choose one sweet and eat it and I choose another sweet. What is the probability that I will choose a yellow sweet then a red sweet?

5. I choose a card from a pack of playing cards, replace it, then I choose another card. What is the probability that I will choose two Kings?

6. I choose a card from a pack of playing cards, keep it, and then I choose another card. What is the probability that I will choose two Aces?

7. What is the probability of rolling two dice and getting a six on one and a five on another?

Independent vs dependent events

Answers:

Independent	Dependent
Tossing a head on a 10p coin and tossing a head on a £1 coin	Choosing a girl for my football team, then choosing another girl
Rolling a six on one die and rolling a five on another die	Picking an ace from a pack, keeping it and picking another ace
Choosing a red ball from a bag, replacing it, then a black ball	Choosing a yellow sweet from a bag, eating it and choosing a red sweet
Choosing a king from a pack of cards, replacing it and choosing another king	

Answers to extension activity:

1. What is the probability of tossing a head on a 10p coin and tossing a head on a £1 coin? $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

2. What is the probability of rolling a *six* on one die and rolling a *five* on another die? $^1/_6 \times {}^1/_6 = {}^1/_{36}$

3. There are 6 boys and 6 girls. I am going to choose five people for my football team. What is the probability of choosing a girl, then another girl? $\frac{1}{2} \times {}^5/_{11} = {}^5/_{22}$

4. A bag contains 4 red sweets and 7 yellow sweets. I choose one sweet and eat it and I choose another sweet. What is the probability that I will choose a yellow sweet then a red sweet? $^7/_{11} \times {}^4/_{10} = {}^{14}/_{55}$

5. I choose a card from a pack of playing cards, replace it, then I choose another card. What is the probability of me choosing two Kings? $^4/_{52} \times {}^4/_{52} = {}^1/_{169}$

6. I choose a card from a pack of playing cards, keep it, and then I choose another card. What is the probability of me choosing two Aces? $^4/_{52} \times {}^3/_{51} = {}^1/_{221}$

7. What is the probability of rolling two dice and getting a six on one and a five on another? $^1/_6 \times {}^1/_6 + {}^1/_6 \times {}^1/_6 = {}^1/_{18}$

4.4 Data handling carousel activity

Plan

I hour	**Carousel of advanced numeracy activities** See list of activities and instructions on below. One activity to be set out per table, tables in cabaret style around the room (if short of space, activities can be placed face-down in the centre of each table until required). Participants move from one table to another to try out the activities, and use review sheet to record comments. Trainer to intervene and provide support with activities where requested. Trainer should emphasise that participants are not expected to be able to do all the activities – the idea is for them to assess which ones they are confident on, which they need to brush up on and which they need a lot of help with – and the Data Handling Self Assessment Record Sheet is to record this.	Data handling activity sheets (a – g) Carousel Review sheet

List of carousel activities

Box and whisker plots (Activity a)
Labelling activity – Laminate diagram on A4 sheet. Laminate label cards and cut out. Laminate answer sheet.

Data handling statements (Activity b)
True/false – Colour photocopy, laminate and cut out cards. Laminate answer sheet

Cumulative frequency (Activity c)
Paper exercise – Laminate graph and answer sheet. Photocopy question sheets.

Pie charts (Activity d)
Matching activity –Laminate cards and cut out. Laminate answer sheet. Photocopy participants' answer sheets.

Statistics terminology (Activity e)
Group discussion – Photocopy a pile of the activity and leave on the table. Laminate possible solution sheet. Provide some counters or blocks.

Stem and leaf diagram (Activity f)

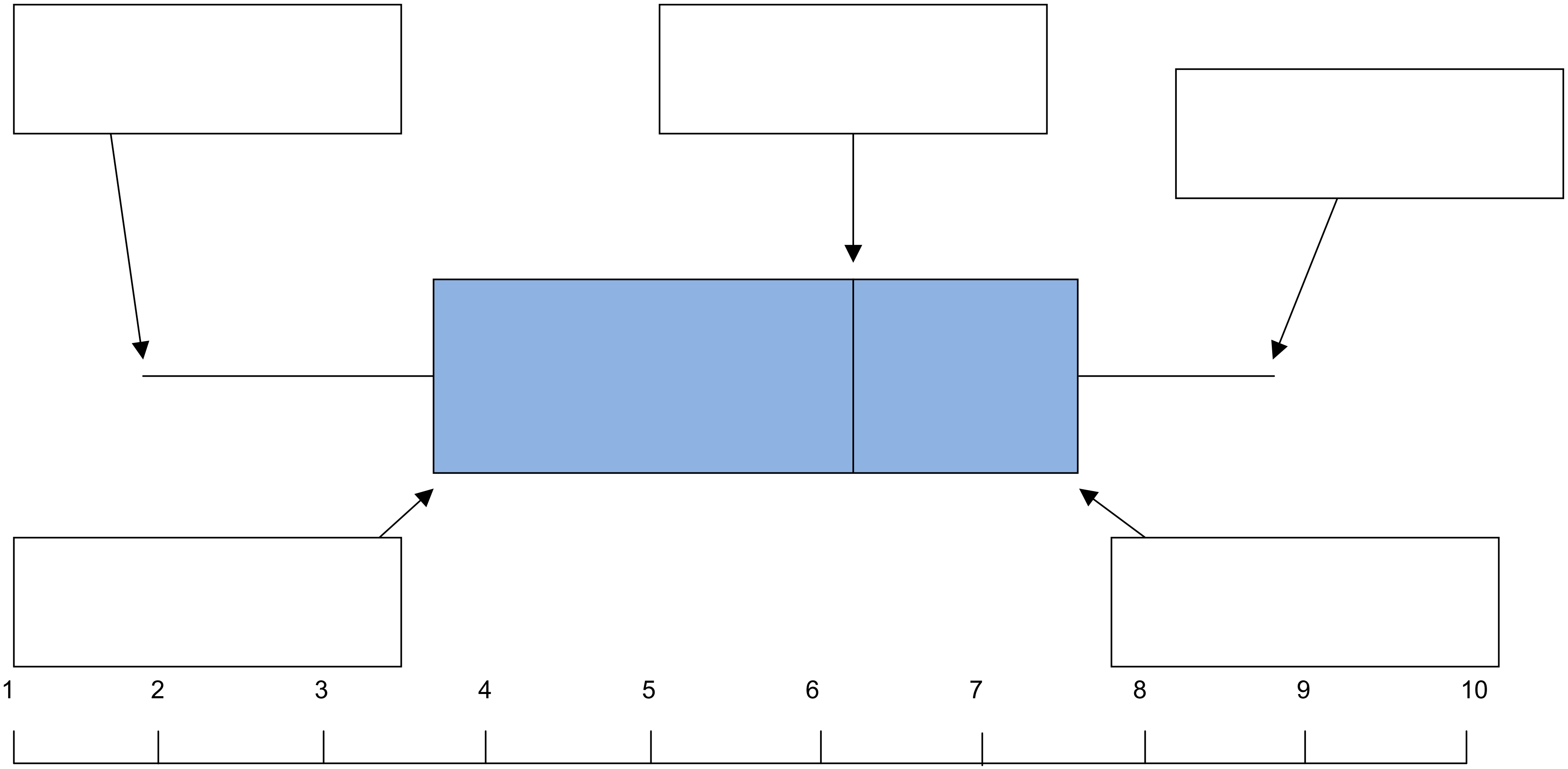
Labelling – Laminate instructions. Provide non-permanent markers. Laminate possible solution sheet.

Working with scale (Activity g)

Group discussion - Laminate instructions and graphs. Laminate possible answers.

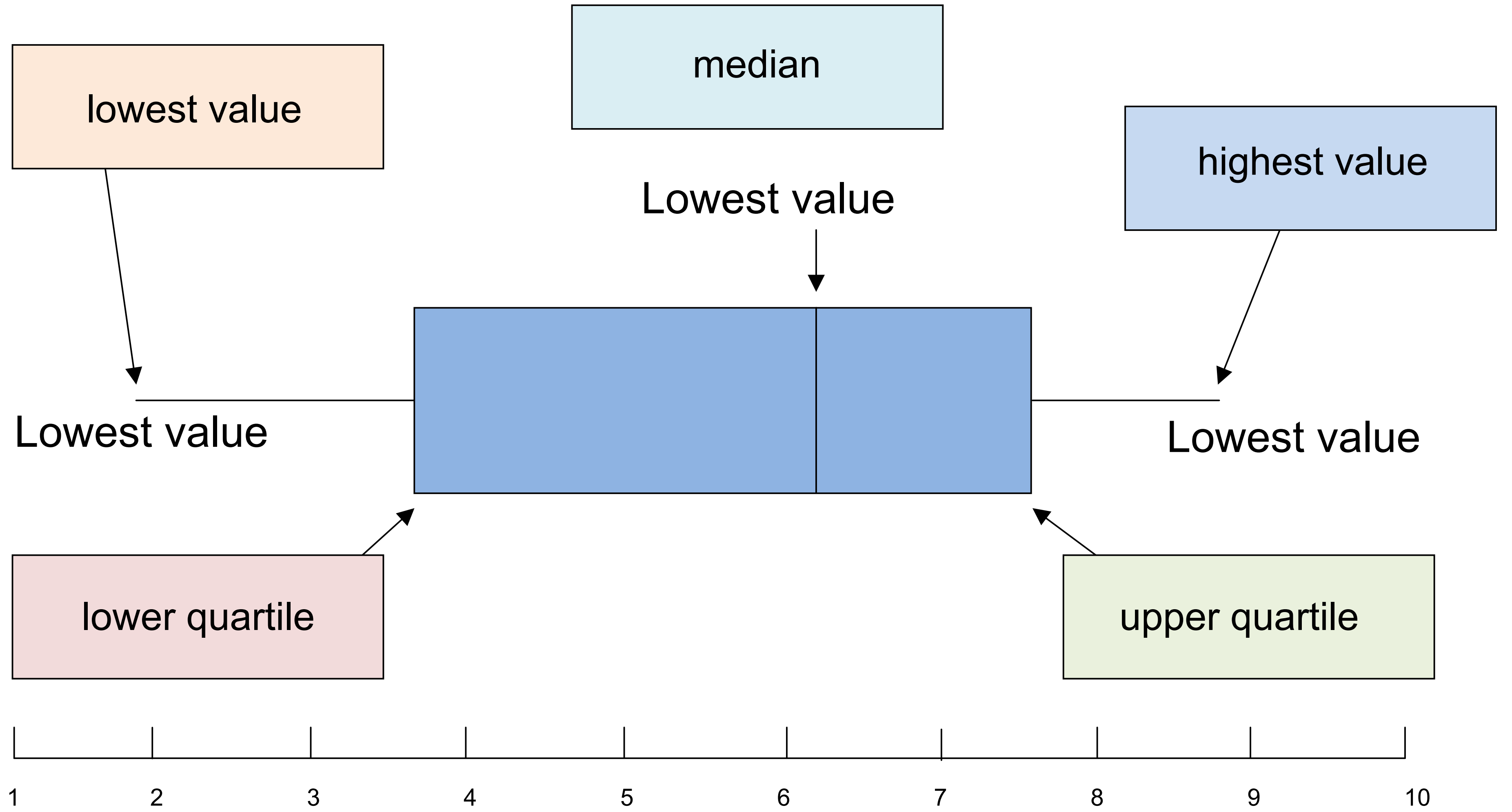
Data Handling Carousel Activity A (to be laminated)

Box and whisker plots



Box and whisker plots

Answer



Handling carousel Activity A (to be laminated and cut into cards)

Box and whisker plots

lower quartile	upper quartile
mean	median
lowest value	highest value

Data Handling Carousel Activity B (to be laminated and cut into cards)

Instructions
Data Handling statements

Separate the cards into 3 sets according to whether you think the statements are

- always true
- sometimes true
- never true

The mean is what you get when you add up all the data and divide by the number of values	The mean is what you get when you add up all the data and multiply by the number of values
The mean is the lowest value in a set of data	The range is the difference between the highest value and the lowest value
The range is the highest value plus the lowest value	The range is the highest value
A pie chart can be used to compare proportions of data	A pie chart is used to see if there is a relationship between groups of data
A pie chart uses symbols to represent data	The mode is the most common value
The mode is the biggest value	The mode is the smallest value

A frequency table shows a line going up and down	A frequency table shows how many of each item you have
A frequency table is a set of bars or columns	The vertical axis is a line that goes up the side of a graph
The vertical axis is a line that goes along the bottom of a graph	The vertical axis is a line drawn diagonally through a graph
A line graph can be used to show how data is changing with time	A line graph shows a line going up and down
The line in a line graph starts at zero	A bar chart is used to compare numbers of items
A bar chart has numbers along the bottom	A bar chart uses a key instead of a scale
A pictogram is a chart that needs a key to show what the symbols mean	A pictogram is a picture of a graph
A pictogram is a chart showing coloured bars	

Data Handling carousel Activity B (to be laminated)

True/false data handling cards

Answers

Always true

- The mean is what you get when you add up all the data and divide by the number of values
- The range is the difference between the highest value and the lowest value
- A pie chart can be used to compare proportions of data
- The mode is the most common value
- A pictogram is a chart that needs a key to show what the symbols mean
- The vertical axis is a line that goes up the side of a graph

Sometimes true

- The mean is the lowest value in a set of data (this would only be true if all the data had the same value)
- The range is the highest value (it is possible that the range could be the same as the highest value i.e. if the lowest value equals zero)
- The mode is the biggest value
- The mode is the smallest value
- A frequency table shows how many of each item you have (not true for grouped frequency tables)
- A line graph can be used to show how data is changing with time
- A line graph shows a line going up and down
- The line in a line graph starts at zero
- A bar chart is used to compare numbers of items
- A bar chart has numbers along the bottom (e.g. a survey of favourite numbers)

Never true

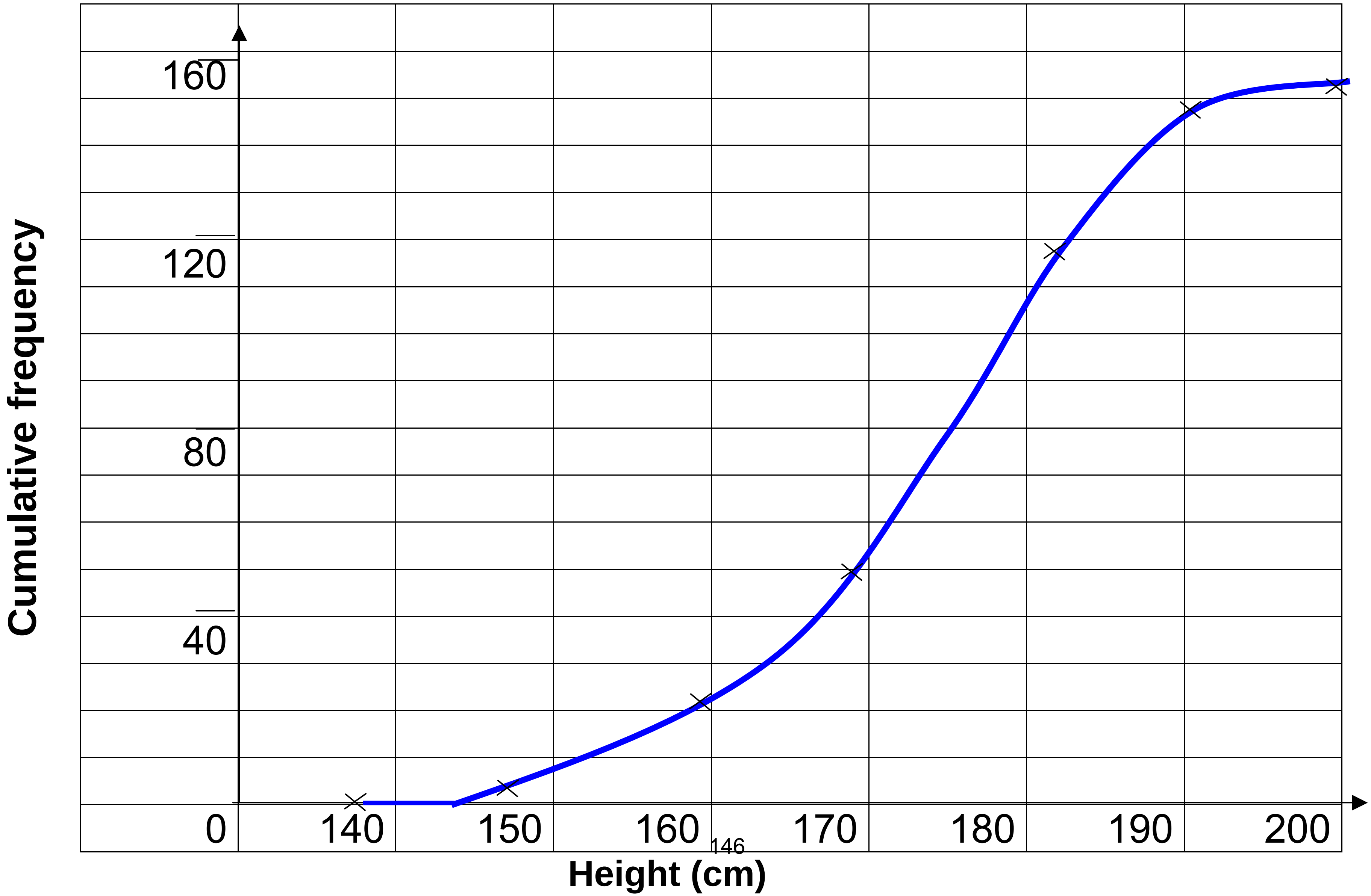
- The mean is what you get when you add up all the data and multiply by the number of values
- The range is the highest value plus the lowest value
- A pie chart is used to see if there is a relationship between groups of data
- A pie chart uses symbols to represent data
- A pictogram is a picture of a graph
- A pictogram is a chart showing coloured bars
- A frequency table shows a line going up and down
- A frequency table is a set of bars or columns

- The vertical axis is a line that goes along the bottom of a graph
- The vertical axis is a line drawn diagonally through a graph
- A bar chart uses a key instead of a scale

Data Handling Carousel Activity C (to be laminated)

Cumulative frequency diagram

Instructions



Data Handling Carousel Activity C (to be photocopied)

Cumulative frequency diagram

The heights of 160 people were recorded to produce this diagram.

Use the diagram to estimate values of:

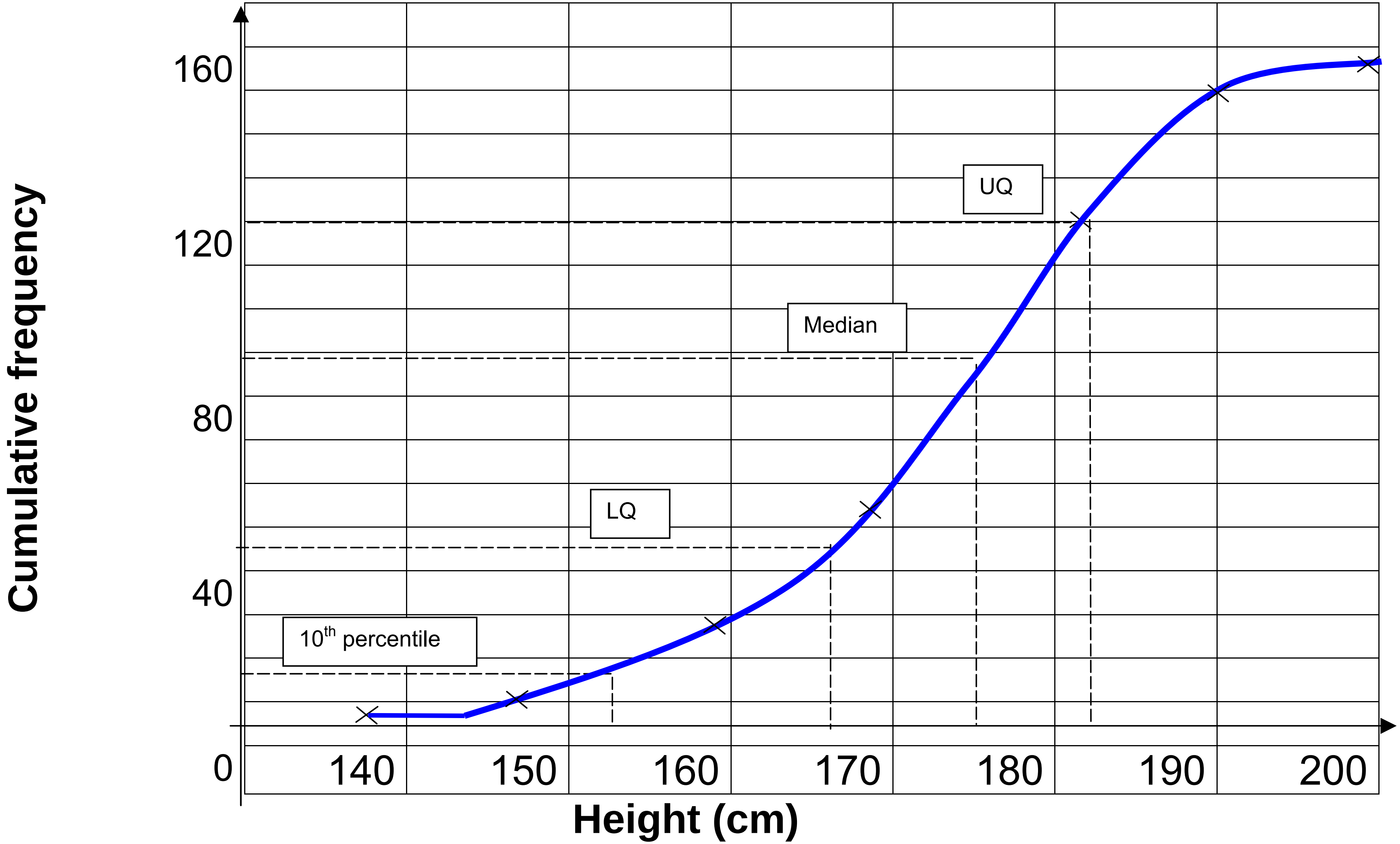
a) 10th percentile
b) Lower quartile
c) Median
d) Upper quartile

Record your estimates in this table:

Statistic	Estimated value
10th percentile	
Lower quartile	
Median	
Upper quartile	

Data Handling Carousel Activity C (to be laminated)

Cumulative frequency diagram - Answers



Data Handling Carousel Activity C (to be laminated)

Statistic	Estimated value
10th percentile	154 - 156cm
Lower quartile	167 - 169cm
Median	176 - 178cm
Upper quartile	183 - 185cm

Data Handling Carousel Activity D (to be photocopied)

Instructions

Pie Chart activity

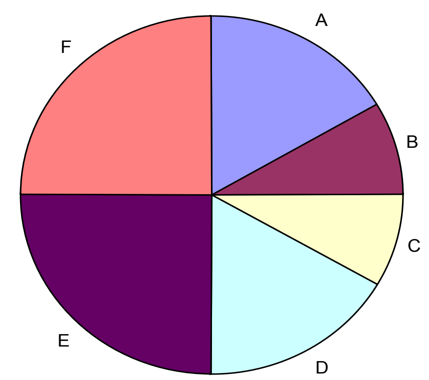
Match the table to the correct pie chart and write in the appropriate sector letter and angle

Check your answers overleaf

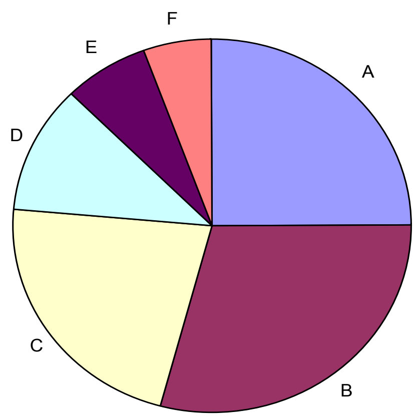
Number	Sector	Angle
8		
10.5		
2.5		
4		
9		
2		

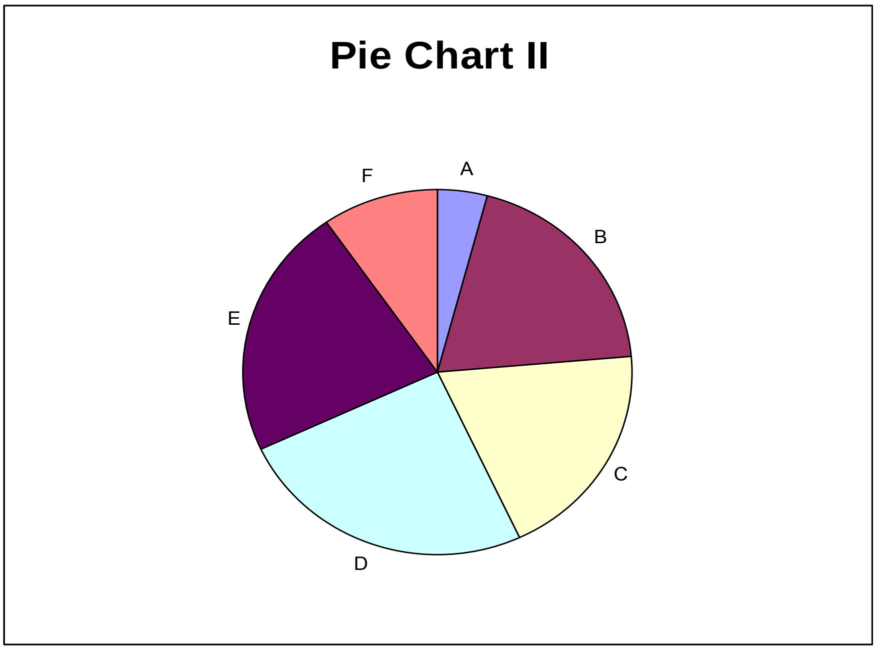
Data Handling Carousel Activity E (to be laminated and cut into cards)

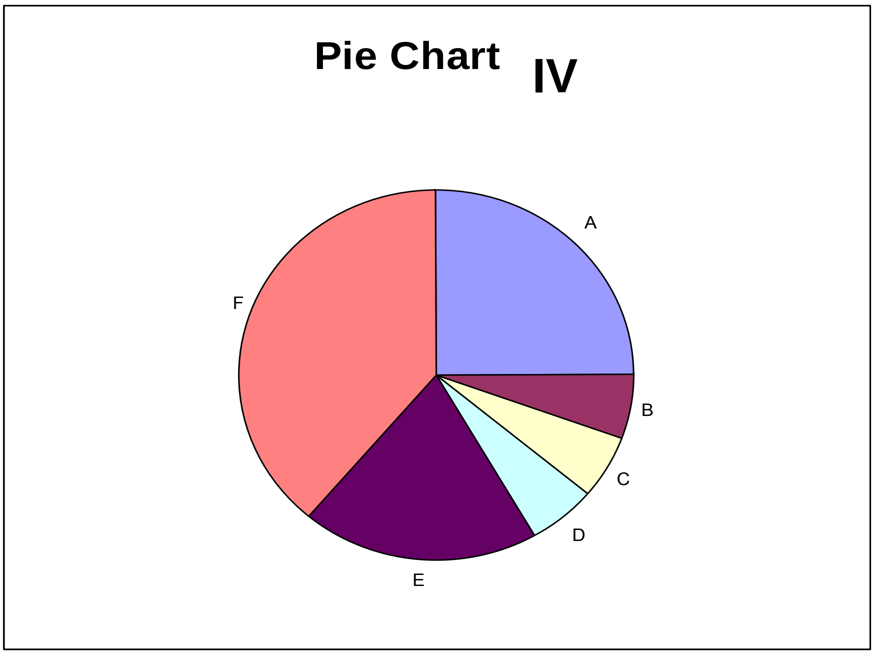
Pie Chart III



Pie Chart I




Pie Chart II
A
B
C
D
E
F


Pie Chart IV
A
B
C
D
E
F

Data Handling Carousel Activity D (to be laminated)

Pie charts
Answers

Pie chart I

Number	Sector	Angle
8	C	80°
10.5	B	105°
2.5	E	25°
4	D	40°
9	A	90°
2	F	20°

Data Handling Carousel Activity E (to be laminated and cut out)

Instructions

Statistical terminology

Match the definitions to the statistical terms.

Check your answers on the answer sheet overleaf

Data Handling Carousel Activity E (to be laminated and cut out)

What is the mode?	The most common value
What is discrete data?	Quantitative data that can only take particular values in a given range
What is qualitative data?	Non-numerical data
What is frequency?	A measurement of the number of times a value occurs
What is quantitative data?	Numerical data
What is continuous data?	Quantitative data that can take any value in a given range
What is a sample?	A group of items taken from the population
What is the mean?	The result that would be given by equal distribution of all values
What is random sampling?	A sampling method giving every item an equal chance of being selected
What is the lower quartile?	The value that falls one quarter of the way through an ordered set of data
What is the median?	The value that falls half way through an ordered set of data
What is the range?	The difference between the highest and lowest values
What is inter-quartile range?	The difference between the upper and lower quartiles
What is standard deviation?	A measure of dispersion of the data equal to the square root of the variance

Data Handling Carousel Activity E (to be laminated and cut out)

Statistical terminology

Answers

What is the mode?	The most common value
What is discrete data?	Quantitative data that can only take particular values in a given range
What is qualitative data?	Non-numerical data
What is frequency?	A measurement of the number of times a value occurs
What is quantitative data?	Numerical data
What is continuous data?	Quantitative data that can take any value in a given range
What is a sample?	A group of items taken from the population
What is the mean?	The result that would be given by equal distribution of all values
What is random sampling?	A sampling method giving every item an equal chance of being selected
What is the lower quartile?	The value that falls one quarter of the way through an ordered set of data
What is the median?	The value that falls half way through an ordered set of data
What is the range?	The difference between the highest and lowest values
What is inter-quartile range?	The difference between the upper and lower quartiles
What is standard deviation?	A measure of dispersion of the data equal to the square root of the variance

Instructions

Stem and leaf diagram

Use the non-permanent marker to create a stem and leaf diagram showing these heights of a group of people:

162cm, 158cm, 170cm, 149cm, 166cm, 163cm, 157cm, 171cm, 168cm.

Write your key in the box provided and record the values of the modal group and range.

Key:

Modal group:

Range:

Data Handling Carousel Activity F (to be laminated)

Stem and leaf diagram

Answers

Use the non-permanent marker to create a stem and leaf diagram showing these heights of a group of people:

162cm, 158cm, 170cm, 149cm, 166cm, 163cm, 157cm, 171cm, 168cm.

Write your key in the box provided and record the values of the modal group and range.

Key: 14 | 9 = 149 cm

14	9
15	7, 8
16	2, 3, 6, 8
17	0, 1

Modal group: 160 - 169

Range: 149 - 171

Data Handling Carousel Activity G (to be laminated)

Instructions

Working with scale

- The three graphs represent the same information. Explain why they look so different.

- Which graph might be used to:

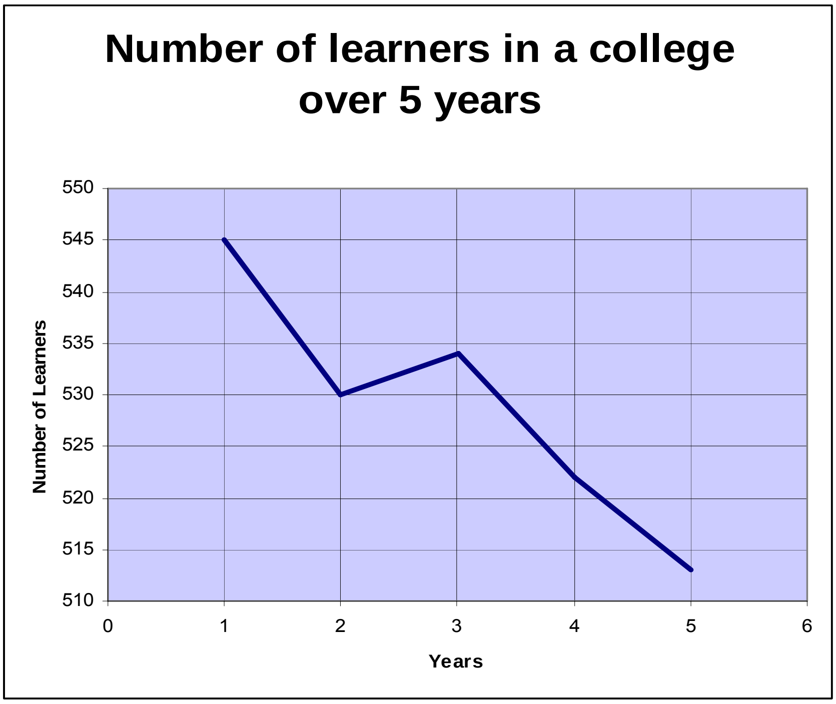
    - argue for the need for redundancies at the college

    - include in a college brochure

- Give another context in which graph A could give a false impression of the facts

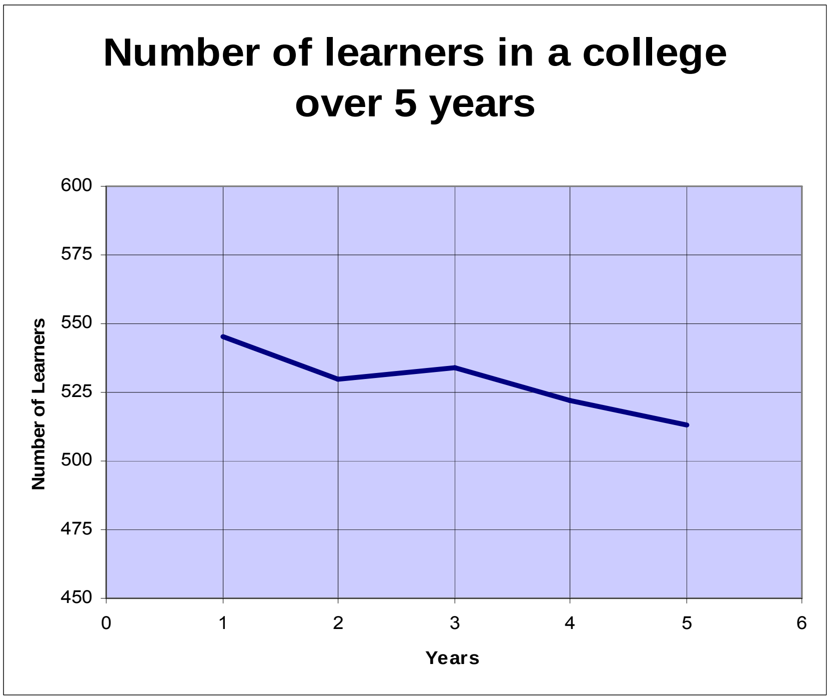
- Give another context in which graph C could give a false impression of the facts

Graph A



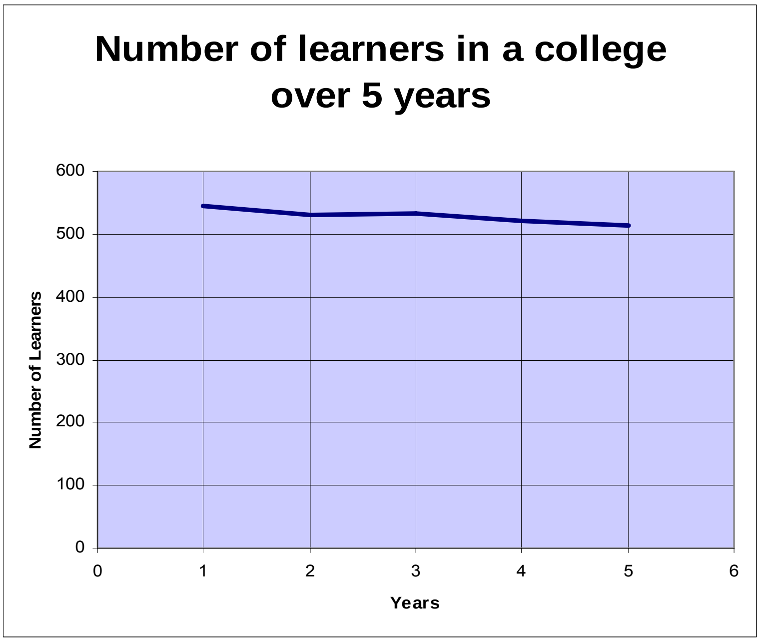
The number of learners we've had in our college has dropped significantly over the last five years

Graph B



There has been a small decrease in the number of learners in our college over the last five years

Graph C



There has been no significant change in the number of learners in the last five years

Working with scale
Answers

- The three graphs represent the same information. Explain why they look so different.
  The scales are different. The actual data only ranges from 513 – 545 learners but the three graphs cover different ranges of data and the scale increments vary greatly.
  Graph 1 covers the data range 510 – 550 and the scale has increments of 5 learners. Therefore, small changes can be made to look more significant.
  Graph 2 covers the data range 450 – 600 and the scale has increments of 25 learners.
  Graph 3 covers the data range 0 - 600 and the scale has increments of 100 learners. Therefore, large changes can be made to look less significant.

- Which graph might be used to:
  - argue for the need for redundancies at the college
    Graph A makes it look as though there has been a huge drop in the number of learners
  - include in a college brochure
    Graph C might be used to disguise the fact that there has been a drop in learner numbers

- Give a context in which graph A could give a false impression of the facts
  A small decrease in prices could be made to look like a large decrease

- Give a context in which graph C could give a false impression of the facts
  A significant reduction in the number of trains on time could be made to look like there has been no significant change

Data Handling Self Assessment Record Sheet

Activity	*How I did on this*	*What else I need to know*
Box and whisker plot		
Data handling statements		
Cumulative frequency		
Pie charts		
Statistics terminology		
Stem and leaf diagram		
Working with scale		

4.5 Data classification activity

This is a whole group activity that requires you to classify a data according to the type of data e.g. whether it is qualitative or quantitative.
You will be given an attribute card – this is an example of a piece of data that might be collected.
Data type cards (Qualitative, Discrete Quantitative or Continuous Quantitative) have been placed around the room. Use blu-tack to place this card under the correct data type card.
Once all of the attribute cards have been classified by the group, have a look at how the cards have been classified. Do you agree with the classification of all the cards or are there any that you would like to question?
You will have a whole group discussion on this.

Attribute cards

Shoe size
Plant growth – stem length
Place of birth
Age in years
Height
Make of car

Foot length
Flower colour
Number of seeds germinating
Weight
Favourite TV programme
Number of children

Answers to data sorting activity:

Data label	Data type
Favourite TV programme	Qualitative
Flower colour	Qualitative
Place of birth	Qualitative
Make of car	Qualitative
Number of seeds germinating	Discrete quantitative
Number of children	Discrete quantitative
Shoe size	Discrete quantitative
Age in years	Discrete quantitative*
Weight	Continuous quantitative
Height	Continuous quantitative
Foot length	Continuous quantitative
Plant growth – stem length	Continuous quantitative

* age in years is regarded as discrete as we are stating it in years rather than allowing the age to take any value in between the years.

Types of data cards

Qualitative Data
Continuous Quantitative Data
Discrete Quantitative Data

4.6 Location and dispersion sorting activity

This is a small group activity.
Your group has been given two sorting sheets – one for location and one for dispersion. You will also be given a set of cards showing various statistical measures. Sort the cards according to whether they are measures of dispersion or location and attach them to the blank boxes on the appropriate activity sheet.
Check your answers using the handout and discuss any differences from the answer sheet in your group. There will be a whole group discussion where you can clarify any misunderstandings or misconceptions relating to this activity.

Answers

Location and dispersion

In statistics a measure of **location** is a single figure which gives a typical or, in some sense, central value for a distribution or sample. The most common measures of location are the *mean*, the *median* and less usefully, the *mode*. For several reasons, the mean is usually the preferred measure of location, but when a distribution is *skew* the median may be more appropriate.

A measure of **dispersion** is a way of describing how scattered or spread-out the observations in a sample are. . . . Common measures of dispersion are the *range*, *interquartile range, variance and standard deviation*. The range may be unduly affected by odd high and low values. . . . The standard deviation is in the same units as the data, and it is this that is most often used.

Reference: *Clapham, C. & Nicholson, J. (2005) Third edition Oxford Concise Dictionary of Mathematics. Oxford, Oxford University Press*

Location

- Average
- Mean
- Median
- Mode
- Central value
- Quartiles
- Percentiles
-

Dispersion

- Spread
- Scatter
- Range
- Outliers
- Variance
- Standard deviation
- Inter-quartile range

Average

Percentiles

Quartiles

Mode

Central Value

Median

Mean

Standard Deviation

Scatter

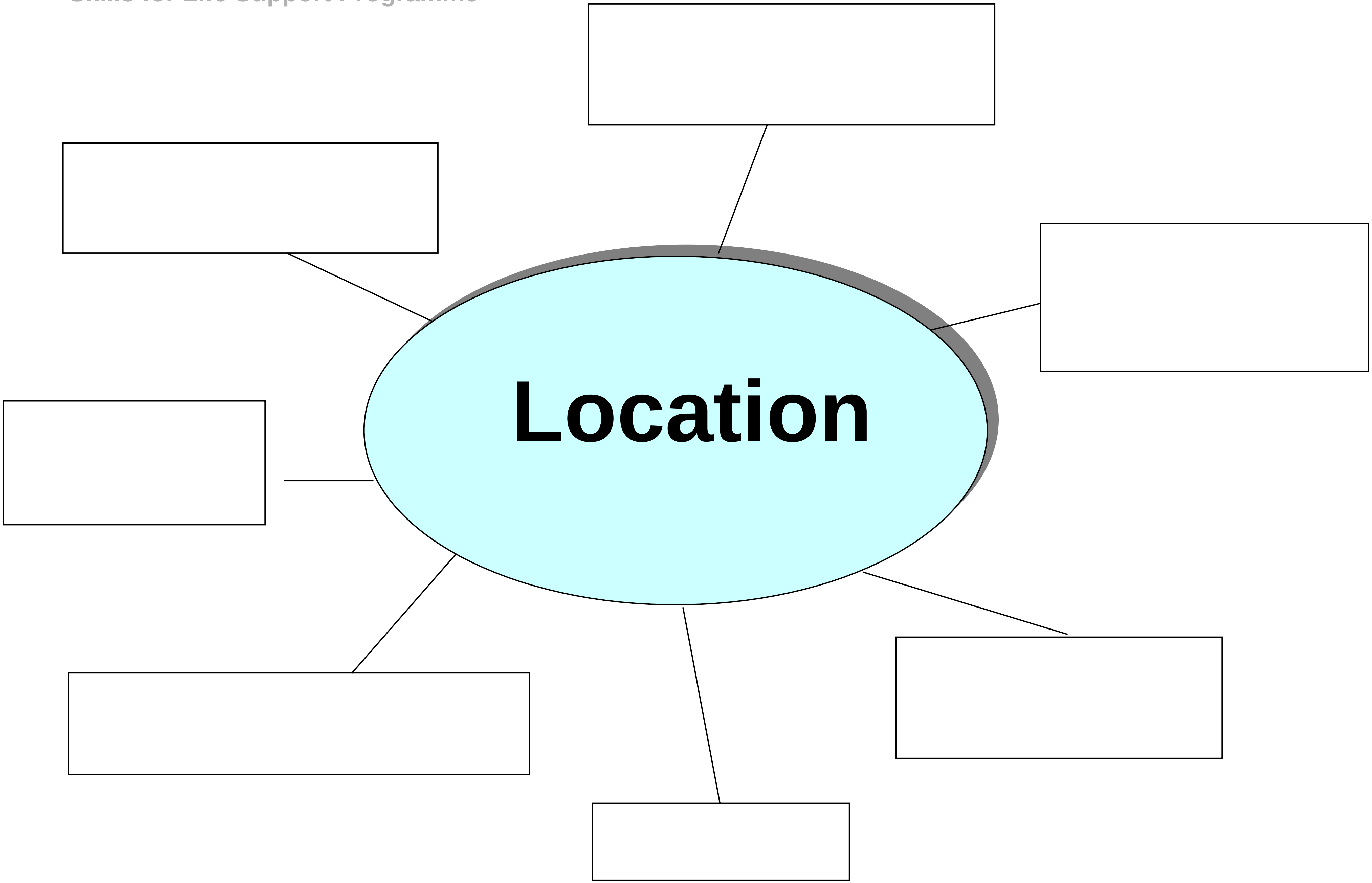
Spread

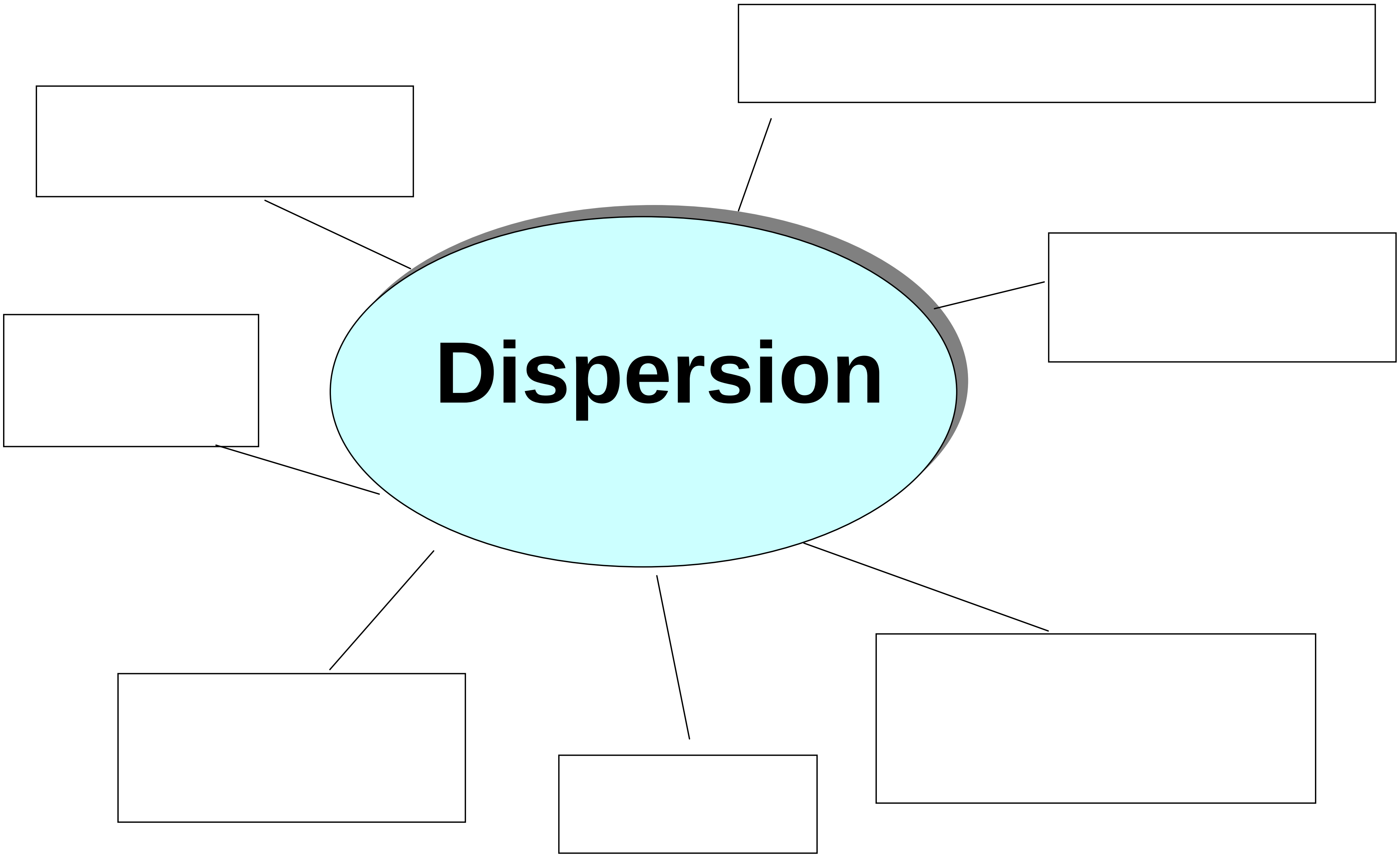
Interquartile Range

Range

Variance

Outliers


Location


Dispersion

4.7 Journey times activity

Use convenience sampling to select 5 journey times from the sheet.
Calculate the mean and median journey time using those 5 pieces of data.

Discuss your results – do you think you have come close to the real results? Was your sampling method fair? Can you think of any bias that may have been introduced into the sampling method?

Extension:
Use stratification sampling to select 5 journey times from the sheet and re-calculate the results.
Are the results different? Which method do you think is best?

Journey times activity

Use cluster sampling to select 5 journey times from the sheet.
Calculate the mean and median journey time using those 5pieces of data.

Discuss your results – do you think you have come close to the real results?
Was your sampling method fair? Can you think of any bias that may have been introduced into the sampling method?
Were there any other problems with using this method of sampling?

Extension:
Use stratification sampling to select 5 journey times from the sheet and re-calculate the results.
Are the results different? Which method do you think is best?

Journey times activity

Use random sampling to select 5 journey times from the sheet.
Calculate the mean and median journey time using those 5 pieces of data.

Discuss your results – do you think you have come close to the real results?
Was your sampling method fair? Can you think of any bias that may have been introduced into the sampling method?
Were there any other problems with using this method of sampling?

Extension:
Use stratification sampling to select 5 journey times from the sheet and re-calculate the results.
Are the results different? Which method do you think is best?

Using a calculator to take a random sample

- Locate and press the random function (may be shown as ran).
  You will probably need to press shift first.
  This should give you a random number between 0 and 1 e.g. 0.915 (you may need to press = to get this to show.

- Multiply by the number of items in the sample.
  e.g. if there are 20 items in the sample, using the example of the random number above (0.915):
  $0.915 \times 20 = 18.3$

- Round the answer to the nearest whole number. This will give you the number of the item to choose for your random sample (18 in this example).

- Repeat until you have the required number of items in your random sample.

Hints on how to take the samples

Convenience sampling: A sample is taken by asking any set of individuals from population. Simply pick any 5 journey times from the sheet.

Cluster sampling: In this technique the population is divided into small subsections. The subsections are chosen at random and then ALL members of the relevant subsection are studied.

In order to end up with a sample of 5 journey times you must divide the population into 20 subsections e.g. 1-5, 6-10 etc. Then randomly select one of these groups of 5 journey times.

Stratification sampling: In this technique one must divide the population into various groups, and then estimate the proportions of the total population which are accounted for by each of the groups.

You could, for example, divide the data into the following groups: car; bus and train; other. You then calculate the proportion each group represents of thee total population and multiply each proportion by 5 to work out how many you need from each group to get a representative share in the sample. You may need to use rounding to ensure the total sample comes to 5. Finally, use random sampling to select that number from each group.

Journey to work times

100 people in a college were surveyed about how long it took for them to travel to the college on a particular day. These were their responses:

Person id number	Mode of transport	Journey time - minutes
1	Car	12
2	Car	14
3	Car	22
4	Car	15
5	Car	18
6	Car	20
7	Car	21
8	Car	30
9	Car	25
10	Car	18
11	Car	22
12	Car	27
13	Car	16
14	Car	10
15	Car	18
16	Car	21
17	Car	28
18	Car	55
19	Car	45
20	Car	18
21	Bus	20
22	Bus	15
23	Bus	19
24	Bus	25
25	Bus	22
26	Bus	32
27	Bus	25
28	Bus	23
29	Bus	25
30	Bus	26
31	Bus	35
32	Bus	42
33	Bus	28
34	Bus	40
35	Bus	35
36	Bus	36
37	Bus	27
38	Bus	50
39	Bus	42
40	Bus	55
41	Bus	45
42	Bus	41
43	Bus	36
44	Bus	41
45	Bus	65
46	Bus	38

47	Bus	33
48	Bus	29
49	Bus	44
50	Bus	40
51	Bus	33
52	Bus	45
53	Bus	52
54	Bus	48
55	Bus	36
56	Bus	25
57	Bus	25
58	Bus	34
59	Bus	44
60	Bus	37
61	Bus	55
62	Bus	50
63	Bus	39
64	Bus	41
65	Bus	47
66	Bus	38
67	Bus	24
68	Bus	31
69	Bus	56
70	Bus	70
71	Bus	42
72	Bus	33
73	Bus	30
74	Bus	35
75	Bus	22
76	Bus	28
77	Train	56
78	Train	75
79	Train	54
80	Train	46
81	Train	65
82	Walk	20
83	Walk	25
84	Walk	26
85	Walk	8
86	Walk	9
87	Walk	21
88	Walk	14
90	Walk	19
91	Walk	22
92	Walk	25
93	Walk	12
94	Walk	10
95	Walk	19
96	Walk	12
97	Walk	16
98	Walk	24
99	Walk	29
100	Walk	17

Answers:

Mean journey time: 31.6 minutes
Median journey time: 28 minutes

	Mean journey time mins	Median journey time mins
car	22.8	20.5
bus	36.7	36
train	59.2	56
walk	18.2	19

4.8 Fill the gap – bias in sampling

With your partner, select from the possible answer phrases to fill the gaps in the statements below. The phrases may be used more than once. Check your completed task against the answer sheet and discuss any differences with your partner. There will be a whole group discussion whereby you will be able to clarify any misunderstandings or misconceptions.

- The variability among statistics from different samples is called ………………………………..

- Increasing the sample size tends to reduce the ……………………………………………………..

- Increasing sample size does not affect ……………………………………………………….

- The bias that results from an unrepresentative sample is called………………………….……….

- Under coverage and non-response bias are examples of ………………………………………

- …………………………………... happens when individuals chosen for the sample are unwilling or unable to participate in the survey.

- ……………………………………… is the bias that results from problems in the measurement process.

- …………………………………….. is a procedure for sampling from a population in which every element of the population has an equal chance of being selected.

- ………………………………………. occurs when some members of the population are inadequately represented in the sample.

- ……………………………………….. is a sampling method that is free from bias but it does not guarantee an unbiased sample.

Answers:

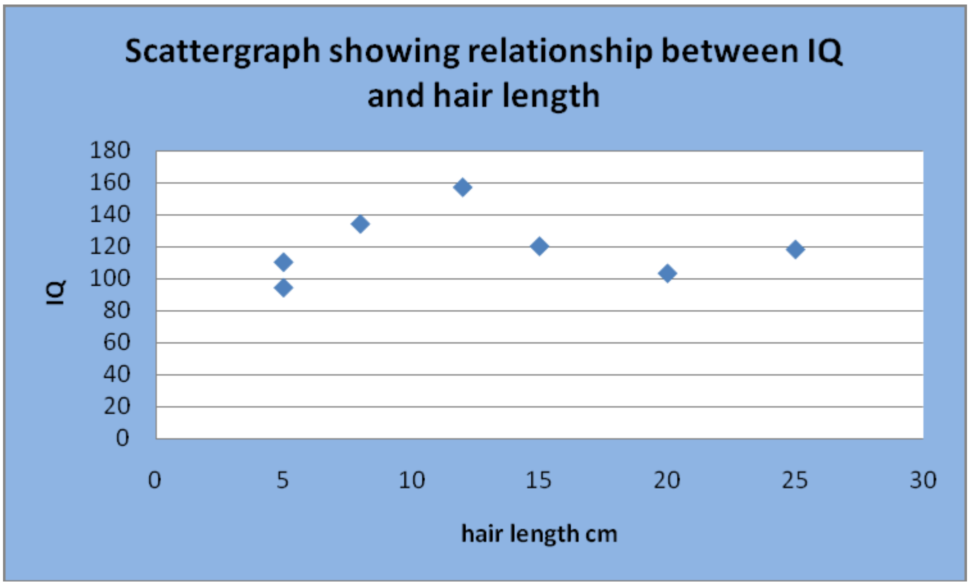
- The variability among statistics from different samples is called **sampling error**.
- Increasing the sample size tends to reduce the **sampling error.**
- Increasing sample size does not affect **survey bias.**
- The bias that results from an unrepresentative sample is called **selection bias**.
- Under coverage and non-response bias are examples of **selection bias.**
- **Non-response bias** happens when individuals chosen for the sample are unwilling or unable to participate in the survey.
- **Response bias** is the bias that results from problems in the measurement process.
- **Random sampling** is a procedure for sampling from a population in which every element of the population has an equal chance of being selected.
- **Under coverage** occurs when some members of the population are inadequately represented in the sample.
- **Random sampling** is a sampling method that is free from bias but it does not guarantee an unbiased sample.

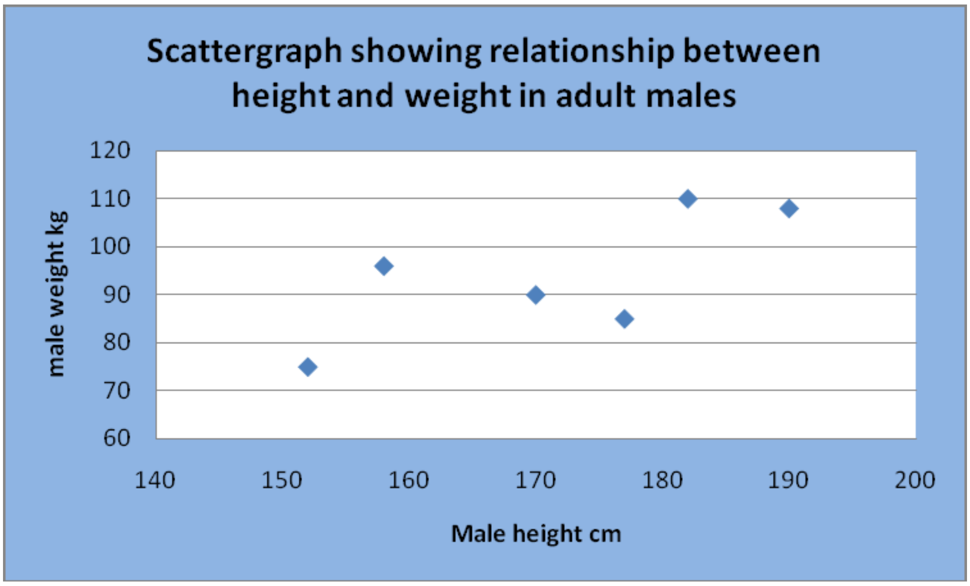
Possible answer phrases

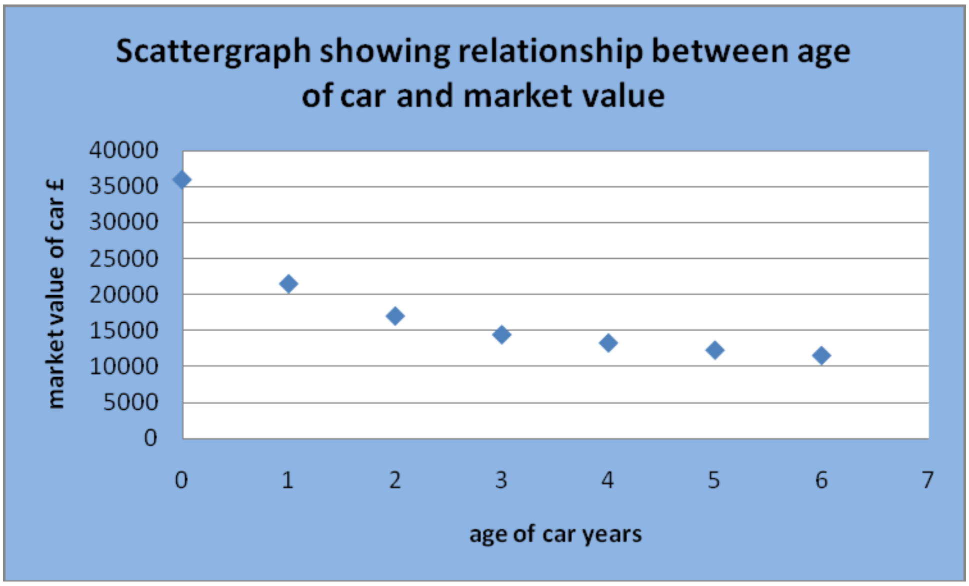
sampling error	survey bias
selection bias	non-response bias
random sampling	under coverage
response bias	

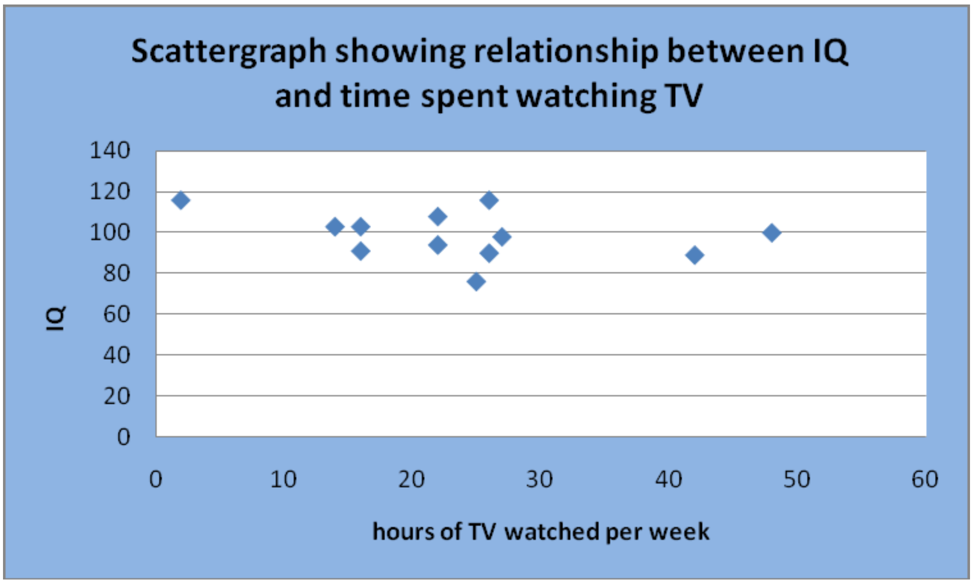
4.9 Correlation cards

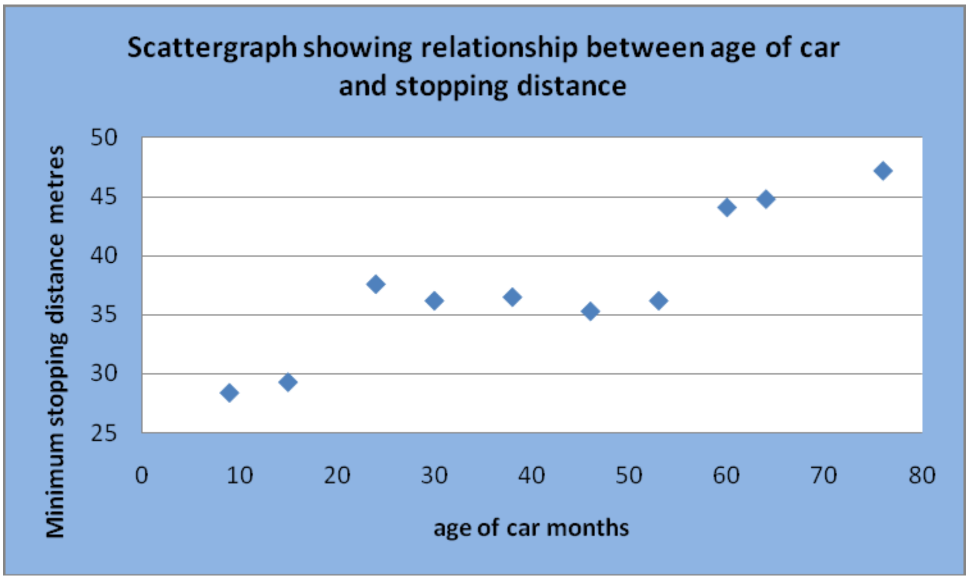
You have been given a set of cards with different scattergraphs showing the relationship between two variables and another set of cards with the Pearson's correlation coefficients for the given scattergraphs.
In your groups, discuss the types of relationship that the scattergraphs show you e.g. whether you think there is a relationship at all; whether the relationship is a strong or weak one and whether any correlation is positive or negative. After your discussion match each scattergraph with one of the Pearson's correlation coefficients.
Check your results with the results of other groups and discuss any discrepancies.
Next, check your results with the answer sheet.
There will be a whole group discussion on correlation whereby you can raise any issues relating to this activity and the topic in general.






Scattergraph showing relationship between age of car and market value
market value of car £
40000
35000
30000
25000
20000
15000
10000
5000
0
0
1
2
3
4
5
6
7
age of car years


Scattergraph showing relationship between IQ and time spent watching TV
IQ
140
120
100
80
60
40
20
0
0
10
20
30
40
50
60
hours of TV watched per week


Scattergraph showing relationship between age of car and stopping distance
Minimum stopping distance metres
50
45
40
35
30
25
0
10
20
30
40
50
60
70
80
age of car months

0.91
0.04
0.62
-0.85
-0.77
-0.36

Answers:

Data	Pearson's correlation coefficient
Age of car and stopping distance	0.91
IQ and hair length	0.04
Height and weight	0.62
Age of car and market value	-0.85
Price of water and distance from attraction	-0.77
IQ and TV	-0.36

Chapter 5
Error analysis

5.1 Introduction to error analysis

Refer to presentation on website
http://www.excellencegateway.org.uk/283499

5.2 Analysing learner errors – number

Learners may make errors in mathematical work they have done for a number of reasons. Below are some possible reasons for learner errors:

- misunderstanding of the problem
- misreading information
- missing an essential step
- lack of knowledge or understanding of a mathematical technique
- forgetting mathematical techniques at the time of the assessment

Can you think of any others?

Some strategies for helping learners with errors

- Use aids and concrete materials such as cube blocks, number lines, place value tables and make references to everyday contexts
- Use inverse operations such as addition to check subtraction calculations and multiplications to check division calculations
- Estimate the answer first where possible such as multiplication of decimals.
- Consider alternative ways of calculation such as partitioning, compensation methods etc.

In the following questions learners have made errors.

- Work out the correct answer.
- Give reasons why you think the learner made the error.
- Can you think of any strategies to help the learners to avoid making these types of errors?

1. Neelam deposits a cheque for £2 567.34
   How much is this to the nearest hundred pounds?
   A £2 566
   B £2 567
   C £2 600
   D £3 000

   Learner's response:

   C £2 567

Number Level 1

2. Sarah makes 40 litres of punch for a party.
To make the punch she mixes juices in the ratio

apple juice : mango juice : orange juice = 3 : 2 : 5

How much mango juice does she use to make 40 litres of punch?
A 8 litres
B 12 litres
C 15 litres
D 20 litres

Learner's response:

D 20 litres

Number Level 2

3. Kyle has room for 52 CDs on his shelf. He already has 38 CDs on his shelf. How many more CDs can Kyle put on his shelf?

Write your answer in the boxes

Show the sum that you do here.

Learner's response:

52
<u>38</u>-
<u>26</u>

Number Entry 3

4. The charge for leaving a boat at a boat yard for six months is £660 + VAT.
VAT is 17 ½ %.
What is the **total** cost of leaving a boat at the boatyard for **a year**?

A £231.00
B £891.00
C £115.50
D £775.50

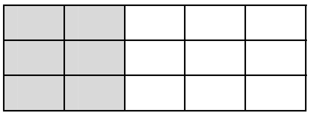
Learner's response:

A £231.00

Number Level 2

5. Shade $^{1}/_{3}$ of the shape below.

Learner's response:



Number Entry 3

5.3 Analysing learner errors – fractions, decimals and percentages

In groups of three, discuss problems, errors, misconceptions that your learners have presented relating to fractions, decimals or percentages.

Each group should identify one key problem and write it on the index card provided.

Swap your card with another group (ensure that the problem is different to the one you identified).

If your group cannot come up with an error or have no previous experience of teaching, use one of the questions on the card.

Discuss the issue on this card:

- Identify the problem, error or misconception
- Identify the underlying cause of the problem
- Suggest strategies to help the learners to avoid making this type of error or demonstrating this misconception.

Record this on the sheet provided.
Your group may want to draw guidance from the successful strategies posted around the room.

Below are some possible reasons for learner errors:

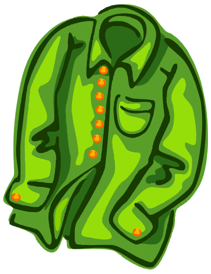
- misunderstanding the problem
- misreading information
- missing an essential step
- not grasping the concept
- lack of knowledge or understanding of a mathematical technique
- forgetting mathematical techniques at the time of the assessment

Discuss your solutions with the group you swapped problem cards with.

Learner errors – cut out and stick on index cards

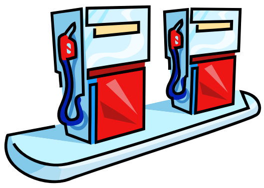
A shirt costs £40.
In a sale it is marked down to £30.
What is the percentage decrease?

Learner's response:
33 $^{1}/_{3}$%



The normal price of fuel is 119.9p for each litre.
A customer has a voucher for '5p off' each litre of fuel she buys.
The customer buys 20 litres of fuel.
How much does she pay?

Learner's response:
£23.88



The table shows all the animals treated by a vet in a week

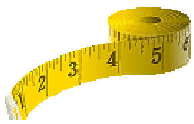
Animals	Cat	Dog	Fish	Bird	Rabbit	Hamster
Number	10	8	4	5	2	1

What fraction of the total number of animals treated were cats?

Learner's response:
$^{1}/_{10}$

Convert 1.75 metres to centimetres

Note: To convert metres to centimetres multiply by 100
Learner's response:
1.7500 cm



5.4 Analysing learner errors – measure, shape and space

Instructions for participants

Look at examples of Level 1 and 2 national numeracy test questions and multiple choice answers.
Investigate the three incorrect answers provided for each multiple-choice and, where applicable, identify the nature of the errors that the writers anticipated a learner might make.
Discuss this in pairs.

5.5 Analysing learner errors – data handling

Learners may make errors in mathematical work they have done for a number of reasons. Below are some possible reasons for learner errors:

- misunderstanding of the problem
- misreading information
- missing an essential step
- lack of knowledge or understanding of a mathematical technique
- forgetting mathematical techniques at the time of the assessment

Can you think of any others?

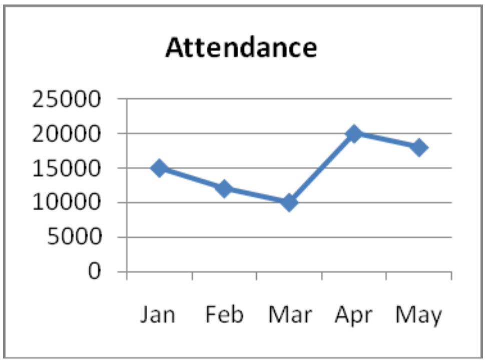
Some strategies for helping learners with errors

- Use aids and concrete materials such as graph and squared paper, blocks, newspapers, magazines and leaflets and make references to everyday contexts
- Check that answers are reasonable and feasible e.g. is the answer within the spread of the data they have been given or outside of it?

In the following questions learners have made errors.

- Work out the correct answer.
- Give reasons why you think the learner made the error.
- Can you think of any strategies to help the learners to avoid making these types of errors?

1. What is the difference between the highest and lowest attendances at the Goodlands Concert Hall?



A 3 000
B 10 000
C 15 000
D 20 000

Learner's response: *C 15 000*

HD Level 2

2. A class exercise is to find out the mean number of sweets in a packet. The number of sweets in different packets are recorded as follows:

14, 12, 19, 16, 14

What is the mean number of sweets per packet?

A 14
B 19
C 15
D 16

Learner's response: *A 14*

HD Level 1

3. What is the range of salaries in the small company below:

Salary	Number of employees
£61 000	1
£32 000	2
£15 000	2
£45 000	3
£21 000	5

A £15 000
B 4
C £40 000
D £46 000

Learner's response: *C £40 000*

HD Level 2

4. A researcher wants to compare height of children with their weight. What is the most effective way to show this from the choices below?

A scatter graph
B line graph
C pie chart
D bar chart

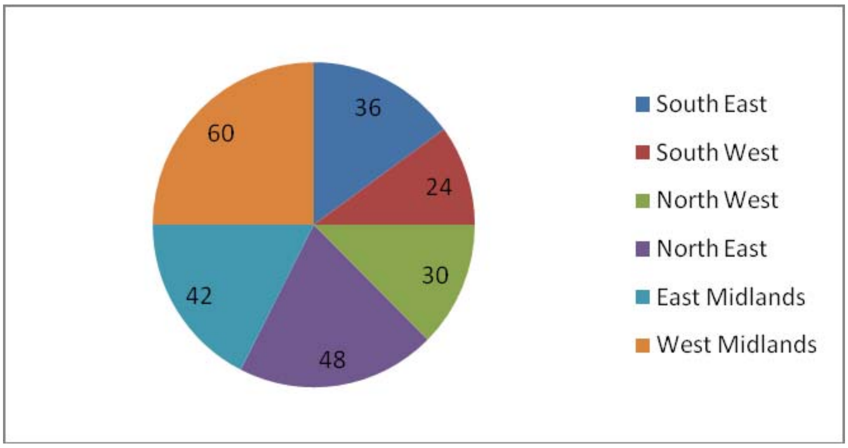
Learner's response:

B line graph

HDLevel 2

5. An event was held for teachers around the country and attendances were analysed by region.
What fraction of total attendances came from the North West?

A $\frac{1}{4}$
B $^{1}/_{6}$
C $^{30}/_{100}$
D $^{1}/_{8}$



Learner's response: *C $^{30}/_{100}$*

HD Level 2

6. The frequency table shows goals scored by a football team this season.

Number of goals	0	1	2	3	4
Frequency	6	10	7	5	1

What is the median number of goals scored?

A 1
B 2
C 4
D 7

Learner's response: *B 2*

HD Level 2

Chapter 6
Teacher notes

6.1 Teacher notes contents

Contents

Chapter 2: Self assessment, number and geometry

2.1 Level 3 Self Assessment carousel

The purpose of the level 3 self-assessment carousel is for participants to identify their own strengths and areas for development in a non-threatening way. This provides an opportunity the trainer to observe individual progress on the different tasks, and also for planning for individual support and setting up PDP/ILPs. Additionally, participants gain an overview of some of the topics they will be studying.

Follow the instructions in the pack.

There are nine activities to prepare for the carousel.

Activity A	Probability questions (GCSE style)
Activity B	What is Pi?
Activity C	Simultaneous Equations
Activity D	Straight Line Graphs
Activity E	Standard Form
Activity F	Cumulative Frequency Curve Worksheet
Activity G	Negative indices
Activity H	Percentages
Activity I	Shapes - Solid geometric shapes should be provided. *See resources list for suppliers.*

In addition, the review sheet and the answer booklet should be provided for participants.

2.2 Checklist of level 3 personal skills
The checklist is provided to be used in conjunction with the carousel for participants to start to plan their learning objectives in their PDP/ILPs once areas to work on have been identified.

2.3 Record of progress checklist
The record of progress follows the self assessment process. The record of progress checklist can help to determine topics that might need more input and to make changes to the scheme of work.

2.4 Number systems and place value

This session has been designed to introduce elements of the history of mathematics showing the contribution of non-western cultures to the subject. Show *The Story of Maths* DVD *Programme 1 The Language of the Universe* (Approximate time: 06:05 – 08:09) and follow with a discussion (see resources list for suppliers). This clip shows the contribution of India and the Arabic world to the number systems we use today, illustrates how some of the different number systems have evolved and the difficulties with a non-place value system.

2.5 Number systems presentation
This activity gives participants an opportunity to attempt to interpret different number systems and begin to see the significance that place value plays in our numbering system.

Give out the Number system and place value handout and A3 copies of the gap filling exercise for participants to attempt in small groups while presenting the slide on the PowerPoint. Follow up by presenting the slide containing the answers.

Participants often have no difficulties with any of the Roman or Egyptian number systems but may struggle with the Babylonian system, in particular the number 74. This is due to the confusion of the representation of the number for 1 and 60 which appears the same. Discuss the fact that in Babylonian times the context would be used to distinguish between them.

2.6 Egyptian multiplication
Egyptian multiplication (also called Russian or peasant multiplication) was a systematic method for multiplying two numbers that does not require the multiplication table and uses only multiplication by 2. The algorithm draws on the binary system millions of years before the invention of computers.

Show *The Story of Maths* DVD – The Language of the Universe (Approximate time 08:35 -10:14). *See resources list for suppliers*.
Display the presentation slides and discuss these with participants. Give out the handout and ask participants to attempt a multiplication using the Egyptian numbers on their own. Provide support as necessary making sure that when drawing up the table, participants use the **powers** of 2 (1, 2, 4, 8, 16, etc) rather than even numbers. Provide calculators so they can check their answers.

2.7 The commutative, associative and distributive laws
This activity introduces participants to some of the laws of arithmetic in preparation for the activity on mathematical modelling.

Laminate the equations and cut into cards making sure you have enough for groups of participants.

Give out the handout and discuss as a whole group giving individual examples for each arithmetic law on a whiteboard. Further examples are in

Haylock D, (2001) Mathematics Explained (*see Chapter 8 Recommended resources).*

Give each group a set of equation cards and ask them to decide whether each is true or false. Prompt participants to substitute numbers if they are unsure how to make a decision. Give groups the answers to check and ask them to produce statements about adding two numbers and adding three numbers.

2.8 Approaches for dividing fractions: modelling exemplar on presentation

The purpose of this activity is to introduce participants to models of mathematical processes. Participants are encouraged to consider using and applying mathematics to the real world rather than mechanically follow an algorithm to get an "answer." The session has been designed to investigate what happens when a fraction is divided by a fraction. Although most numeracy teachers may be familiar with the process, many may not have considered why the process works.

For this session, you will need a variety of numeracy resources such as fraction/percentages blocks and fractions circles. Group the participants so that, where possible, there is at least one person with previous numeracy teaching experience. Give each group a few sheets of flipchart paper and a variety of coloured pens.

Present the slides showing an example of an approach, a worked example and the associated model for **multiplying a fraction by a fraction**. Present the division of a fraction question and ask the groups to suggest an appropriate approach and model. Encourage the groups to make use of physical resources and write their results on flip chart paper or similar. Take feedback from each group and discuss. Show the examples in the presentation.

2.9 Pythagoras' Theorem presentation

This interactive presentation demonstrates the meaning of the famous theorem accredited to Pythagoras in a visual manner. Participants are often familiar with the theorem and may be able to describe the result as "*In a right angled triangle the square of the hypotenuse is equal to the sum of the squares of the other two sides."* Sometimes they can quote

$$a^2 = b^2 + c^2$$

without necessarily understanding the theorem itself.

Show the presentation and discuss what it illustrates. For example, you could ask 'what does the presentation demonstrate?' or 'why is this not a proof?'

GCSE mathematics textbooks can be used to provide many examples of Pythagoras' theorem calculation practice.

2.10 Discovering ratios in right-angled triangles
This activity introduces trigonometry to participants by demonstrating how the trigonometry ratios were derived using discovery learning. Participants could then be shown how to use the formulae to perform trigonometric calculations. The participants will need protractors, rulers, scientific calculators and A3 paper. You may find they get better results by using a sharp 2H pencil.

Photocopy the handouts onto A3 paper.

Put the participants into three groups. Triangle A offers the most guidance, Triangle B has intermediate support, with Triangle C being the most open ended. Group the participants according to their confidence with geometry and the construction of triangles.

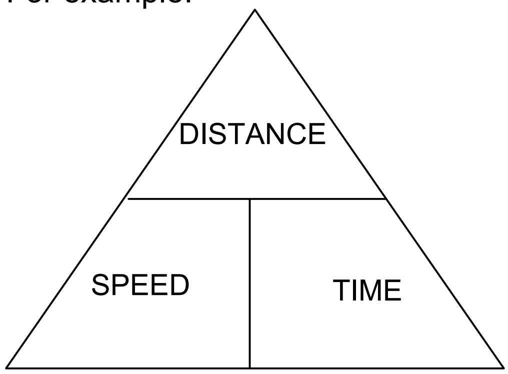
Give each group their instructions and equipment and allow them to work together to carry out the constructions. Ask them to complete the table. Explain that the ratio is usually expressed as a decimal since often participants expect the number to be written as a conventional ratio.

When the groups complete the task, discuss the results showing the answers. Show *The Story of Maths* DVD – *The Language of the Universe* (Approximate time 29:00 - 31:33) and follow with a discussion. *See resources list for suppliers***.**

2.11Trigonometry
This handout is used in conjunction with board work to introduce trigonometry calculations. The activity raises issues of the use of mnemonics and memory in learning and teaching.

The formula triangle that is often used with speed/distance/time calculations can be used in the same way to help participants to remember these relationships.

For example:



Put your finger over the letter you are trying to find, and the triangle tells you what calculation to do for the remaining two letters.

The use of one of the many trigonometry mnemonics may also help participants with remembering the formulae:

"**S**ome **O**fficers **H**ave **C**urly **A**uburn **H**air **T**ill **O**ld **A**ge"

$S = \frac{O}{H}$ $C = \frac{A}{H}$ $T = \frac{O}{A}$ **or** $\text{Sine} = \frac{\text{Opp}}{\text{Hyp}}$ $\text{Cos} = \frac{\text{Adj}}{\text{Hyp}}$ $\text{Tan} = \frac{\text{Opp}}{\text{Adj}}$

GCSE mathematics textbooks can be used to provide many examples of trigonometry calculation practice.

Section 3: Algebra and its applications

3.1 Kinaesthetic expressions

This activity demonstrates the use of a kinaesthetic starter in an algebra context. While engaging in this activity, the participants will be performing several substitutions.

- Give one card to each participant. Trainers may want to differentiate by giving expressions that are more complex to participants who may prefer the challenge (e.g. $\sqrt{n}$).
- Give participants a variety of values for n, for example, n= 1, n= 2, n= 0, n= $^{1}/_{2}$
- Ask participants to line up in order of size (smallest first).
- Where the answers are the same, the participants will be side by side.

This activity encourages participants to discuss the value of the expressions when the values of n are substituted.

Examples:
When n= 1, the order will be

0	**$^{1}/_{3}$**	**$^{1}/_{2}$**	**1**	**2**	**3**	**4**
n - 1 n^2 - 1 n^2 - n	$\frac{1}{n+2}$	$\frac{1}{n+1}$	n^2 $\sqrt{n}$ n^3 n^{-1} ½ (n + 1) $\frac{n+1}{2}$	n^2 +n n^2 +1	3n	4n 2(n + 1)

The order changes for different values of n.

When n = 2, the order will be

0.25	0.33	0.5	1	1.141	1.5	3	4	5	6	8
$\frac{1}{n+2}$	$\frac{1}{n+1}$	n^{-1}	n - 1	√n	½ (n + 1)	n^2 - 1	n^2	n^2 +n	2(n + 1)	4n
					$\frac{n+1}{2}$			n^2 + 1	3n	n^3

3.2 Algebra carousel
The purpose of this self-assessment carousel is for participants to identify their own strengths and areas for development with reference to algebra. Since there are only five activities in this carousel, it has been developed in conjunction with written exercises on fundamental algebra.

Split the participants into two groups. One group works on the written material while the others, in pairs or threes, work through the five carousel activities. The groups can then swap over.

Follow the instructions in the pack.

There are five activities to prepare for the carousel.

- Expressions and equations: true /false (Activity a)
- Matching linear and quadratic graphs (Activity b)
- Real life graphs (Activity c)
- Formulae matching (Activity d)
- Linking algebra with number (Activity e)

3.3 I think of a number activity
This activity provides the participants with practice in writing expressions and equations and directs them towards solving equations using the method of reverse operations.

Start by demonstrating an example of finding a number without mentioning algebra; for example: I am thinking of a number, I double it and add 10, I now have 50. What number was I thinking of?

Then introduce algebra by using a letter for the number and writing each step algebraically on the board. Demonstrate how they can use reverse operations to unpick the equation through reversing the steps of the puzzle and that they will arrive back at the original number. E.g. 50 take away 10 = 40, divide by two = 20.

Take the group through more complex examples, if appropriate, then ask the participants to work in a pair with someone of similar skills level in algebra. Pairs can then work with the examples that are more or less complex.
Give each pair the example sheet and one of the participants a 'think of a number' sheet. Ask them to each come up with an equation that needs solving and to write it down but not to show it to their partner. They then take turns to present the equation to their partner as a think of a number puzzle. For example, I am thinking of a number, I double it and add 10, I now have 50. What number was I thinking of? While they are doing this, the partner should record each step on the sheet (using a separate line in the table provided) and try to solve the resulting equation. The person who came up with the equation should check each step of the solution and the final solution.

They could also discuss methods used to solve the equations (you may need to encourage the participants to value each other's methods rather than thinking there is a 'right way' to solve equations). Partners should support each other with any errors that may arise. Pairs should continue with this activity until they are comfortable with writing and solving equations at a level suitable for them.

3.4 Matching expressions
This activity is designed to support participants with converting written problems to algebraic ones. The matching activity provides a less threatening mode of working on a topic that some may find difficult. The questions are similar in style to those that might be found in a GCSE paper and attempt to bridge the gap between abstraction and 'real life' by combining 'real life' problems with unknown quantities represented by letters.

The activity could be introduced by using a 'real life' example; for example, a boy is 12 years old, how old was he 2 years ago. Ask the trainees how they came up with the solution, and then ask them to write down the solution replacing the 12 with a letter to represent an unknown age.

Ask the trainees to work in small groups and give each group a set of written problem cards and a set of algebraic expression cards.
Once they have matched the sets of cards, hand out the answer sheet for self-checking and engage the whole group in discussion on any issues that may arise.

This activity could be followed by asking groups to set their own written problems and matching expressions for another group to match.
Trainees who are still struggling with writing expressions should be encouraged to study further on this topic.

3.5 Simultaneous equations - ordering steps
This activity is designed to support participants with solving simultaneous equations by breaking the process down into small steps. You will need to discuss simultaneous equations and methods for solving them before giving trainees this activity.

Trainees should work in small groups or pairs and be given the instruction sheet and a set of blue and green cards. They should first order the blue cards, and then match them with the green cards to solve the equations. Discussion of alternative solutions and ordering should be encouraged.

Present the answer sheet for self checking. This could be followed up with asking the pairs or groups of participants to write their own steps on cards for solving given simultaneous equations and swapping with another group to see if they can order the steps and solve the equations.

3.6 Writing equations and solving
This activity is designed to build on the skills acquired from the previous two activities (3.4 and 3.5) in order to write two simultaneous equations and solve them. It is an activity designed to give more practice in the skills of writing and solving simultaneous equations.

Ideally, trainees should work on this activity in groups so that they can discuss the activity and support each other. The equations in this activity can be quite challenging and this may therefore be used as an optional extension activity as appropriate.

Section 4: Data handling

4.1 Probability bingo
This activity is intended as a starter activity that will help you to assess the level of prior knowledge of probability in the group and as a gentle introduction to theoretical probability.

Ensure that everyone is familiar with the game of bingo and playing cards. If anyone is unfamiliar with playing cards then it may help to present the key details. This could include details such as there are 52 cards in a pack; a pack is divided into 4 suits – 2 red: diamonds and hearts and 2 black: clubs and spades; each suit goes from ace (1) to 10 followed by 3 picture cards – jack, queen and king.

Print off two copies of the answer sheet; laminate one copy and cut into individual question cards, without the answers.

Place participants in small groups or pairs. Give each group a bingo card and a highlighter.

Place question cards face down and spread them out. Then invite the groups to take turns to select and read out their question card. Ask groups to highlight the answer if they have it on their card. Continue until one of the groups has highlighted all of the probabilities on their cards. Discuss the answers with the group and review any that they are not comfortable with.

4.2 Probability scale

The purpose of this activity is to ensure that participants have a good understanding of probability scale and the language associated with probability in everyday use.

Enlarge the scales to A3 size and print. You will also need blu-tack or similar.

Print off and laminate one set of the probability term cards for the whole group and a set of event cards for each small group of 3-4 trainees.

Make a large copy of the probability scale and place it on the wall.

Start with a discussion about when probability is used in everyday life and ask for examples of terminology that might be used, such as ‘likely’.

Spread out the probability term cards and ask participants to take a card and place it on the scale where they think it should go. Continue until all the terminology cards have been used. Lead a discussion about where cards have been placed and whether everyone agrees.

Check that ‘always’ and ‘certain’ have been placed at 1 and ‘never’ and ‘no chance’ are at zero. Otherwise emphasise that the placings are subjective within a reasonable range. A discussion of how precise one can be may be useful.

Next, place the participants in groups of 3-4 and give each group a scale and a set of event cards. Ask the groups to place the events where they think they should lie on the scale. Ask them to add their own events on the blank cards. Inform them that they should check what other groups have done once they have placed all of their cards.

Finish with a whole group discussion on any differences in placings. Compare the placing of the event cards with the placing of the probability term cards. Ask participants if they would consider changing the position of the event cards in the light of the comparison with the position of the terminology cards.

Ensure they understand that very few events in life are impossible or certain and that their learners will also find this concept difficult.

4.3 Independent vs dependent events

This activity introduces the concept of independent and dependent events in probability and helps to assess whether participants understand these concepts. Start with a discussion about dependent and independent events.

Use an example to illustrate the point; e.g. decide on a factor you could use to sort people in your group into different sub-groups, e.g. gender. Give everyone in the group a card with a different number on it. Randomly select a numbered card from a set of matching cards and record if the person with that number was male or not. Work out the probability of selecting a male from the group. Replace the numbered card you selected and take a second numbered card. Record whether the second person chosen is male or not. Again,

calculate the probability of selecting a male from the group. Discuss whether the gender of the first person selected had any effect on the probability of the second person selected being male.
Repeat the activity, but this time do not replace the first numbered card selected. Work out the probability of the second person selected being male. Discuss how and why the probabilities have changed in this case. Ask which of the events were independent and which were dependent. Ask trainees to work in small groups. Give each group a set of cards and ask them to sort the cards into independent and dependent events.

Hand out answer sheet for them to self-check and discuss how they found the activity. Hand out the optional question sheet for participants as an extension activity.

4.4 Data handling carousel
The purpose of the data handling self-assessment carousel is for participants to identify their own strengths and areas for development in data handling. The carousel provides activities that cover a range of topics within data handling that trainees should work on via self study if they find they identify any areas for improvement.

Follow the instructions in the pack.

There are seven activities to prepare for the carousel.

- Box plot activity (activity a)
- Cumulative frequency activity (activity b)
- Pie chart activity (activity c)
- Stem and leaf activity (activity d)
- True false data handling activity (activity e)
- Working with scale activity (activity f)
- Statistics vocabulary activity (activity g)

In addition, the review sheet and the answer booklet should be provided for participants. Ask trainees to complete the review sheet and use it to add personal maths skills objectives to their PDP/ILPs.

4.5 Data classification
This activity is intended to provide a gentle introduction to data.

Cut up the data cards and place them on the walls around the room. Laminate and cut up the attribute cards.

Start with a discussion about what ‘data’ means and what kinds of data are collected in life. Point out the types of data cards on the walls and ask trainees to define qualitative and quantitative data.

Hand out attribute cards to individuals or pairs (according to group size) and ask them to stick them under the appropriate data type heading.
Once all cards have been placed, ask trainees to check where all the cards have been placed and to question any they are not sure about.

Discuss any that are wrongly placed and re-check for understanding of qualitative and quantitative data (discrete and continuous). Discuss why it is important to identify whether data is qualitative or quantitative data.

4.6 Location and dispersion sorting

This activity is designed to stimulate discussion around the concepts of location and dispersion and related terminology. It will also allow the assessment of prior knowledge relating to this area.

Participants should work in small groups and you will need one set of materials for each group. Provide groups with laminated location and dispersion cards and a set of the terminology cards. Ask them to sort the cards according to whether they are location or dispersion terms and stick them to the appropriate card. Encourage discussion around the topics involved.

Once the trainees have sorted all of the cards, provide the answer sheet for self-checking. Lead a whole group discussion on any issues that have arisen.

4.7 Journey times

This activity is designed to introduce the participants to a range of different types of sampling methods and to form the basis for a discussion about sampling in general, including what a random sample really means and how samples may be biased.

Start with questions about what sampling is and why we need to do it. Check participant understanding of random sampling and whether they know how to use a calculator to generate a random number – hand out the sheet on how to use a calculator to take a random sample as required. Move on to a discussion around different sampling methods: convenience, cluster and stratification. Ask if they know of any other sampling methods.

Place participants in small groups and give each group a different task sheet (there are three different task sheets). Provide each group with a calculator, hint sheet on how to take the samples and the journey times data. Ask the groups to focus on the main task and only to try the extension task if they have discussed the main task fully.

Take feedback from the groups and discuss the results, particularly any differences between samples. Emphasise the fact that samples would normally need to be much larger than the ones in the activity in order to try to provide a representative sample of the population but inform them that although random sampling as a method is free from bias it does not guarantee that the sample will be a good representation of the population.

4.8 Bias in sampling - Fill the gap
This activity is designed to promote a better understanding of bias in sampling and should be completed after the journey times activity.

Trainees should complete the exercise in pairs. Provide each pair with the fill the gap sheet and the possible answer phrases cut into strips. The 'fill the gap' sheet could be laminated and the trainees could use a non-permanent marker to complete it so that any errors could be corrected easily, after discussion and analysis of the error.

Ask the pairs of trainees to use the phrases to fill the gaps, emphasising that phrases may be used more than once. Encourage them to discuss the various options as they do the activity. Hand out the answer sheet and have a whole group discussion on any issues arising from the activity. Clarify any further misunderstandings / misconceptions.

4.9 Correlation cards
This activity is designed to promote an understanding of the concept of correlation. You may want to preface this activity with a practical activity such as measuring heights and weights of group members, plotting them on a scatter graph and using Excel to calculate Pearson's correlation coefficient.

It would be useful to follow with a discussion around what the results mean, whether participants think that height and weight are related and if other factors are involved.

Follow this up by asking trainees if they can think of any other data that might be related. Steer the conversation toward negative as well as positive correlation. You could ask the trainees to represent what they think the correlation would look like by giving them a piece of laminated graph paper and some sticky spots to represent the data points.

For the correlation card activity, ask the trainees to work in small groups. Each group will need a set of graph cards and a set of correlation coefficient cards. Ask them to match the graph with the appropriate correlation coefficient. Suggest groups check each others' results before handing out the answer sheet for self-checking.

Lead a whole group discussion on any issues arising. It is helpful to point out that correlation does not necessarily imply any particular causal link between the variables.

Section 5: Analysing learner errors

5.1 Introduction to error analysis
One of the most significant changes that has been made in mathematics / numeracy education is to value errors as an opportunity for learning. Display the presentation and discuss the issues raised.

5.2 Analysing learner errors- number
This is the first of two activities designed to help participants with error analysis based on the number curriculum area. The questions given in this activity are typical of exam questions at entry to Level 2. Participants should work in groups. Give out the handout which has a reminder of some of the problems and strategies taken from the presentation.

Where possible, give each group one problem to focus on and feedback to the whole group. Encourage participants to discuss the issues in the problem and make sure they have identified possible strategies. Take feedback from the groups.

5.3 Analysing learner errors - fractions, decimals and percentages
This activity is more suitable where the participants already have experience of teaching numeracy. It is designed to help those experienced teachers to share and discuss with the group examples of learner errors they have come across.

Get the participants working in groups of three making sure there is one member of the group who already has teaching experience. Ask them to discuss problems, errors, misconceptions that their learners have presented relating to fractions, decimals or percentages. Each group should identify one key problem and write it on the index card provided. Swap the card with another group.

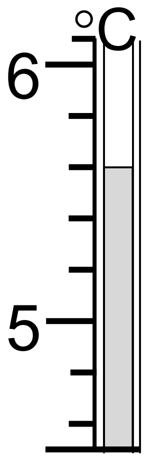
It would be useful to produce your own cards with examples for use when groups are unable to produce their own or have no previous experience of teaching. Ask the participants to record on an A3 sheet and post around the room.

5.4 Analysing learner errors – measures, shape and space
The activity here uses the fact that multiple choice questions are designed so that alternative (incorrect) responses are based upon standard errors made by a number of learners. This makes such questions a useful resource in considering errors and misconceptions.

Collect some measures, shape and space Adult Numeracy level 1 and 2 multiple choice questions. If your participants are currently teaching, you could even ask them to bring in sample exam questions. Exam papers are also available on various websites. Make sure you copy them within the copyright laws.
AQA Exam papers http://web.aqa.org.uk/qual/elc/numeracy_assess.php

An example of the type of MSS question you could choose is given below:

A student working on a catering course checks the temperature in a chill cabinet.
The diagram shows the reading on the thermometer.



The reading on the thermometer is
A 5.3°C
B 5.5°C
C 5.6°C
D 6.4°C

MSS Level 1

Prepare possible response to give to trainees. An example to the above question is given below:

Possible responses

- **A. 5.3:** The learner has assumed that each graduation represents 0.1 unit when it represents 0.2
- **B. 5.5:** The learner has inaccurately read the scale and visually seen that it is about half way up.
- **C. 5.6:** Correct answer
- **D. 6.4:** The learner has read the scale downwards.

An extension to this activity would be to ask participants in their group to come up with strategies to support learners with dealing with these types of questions

5.5 Analysing learner errors – data handling

This is activity is designed to help participants with error analysis based on the handling data curriculum area. The questions given in this activity are typical of exam questions up to Level 2.

Participants should work in groups. Where possible, give each group a different problem card to focus on and feedback to the whole group. Groups who complete this can be given another problem card to work on.

Encourage participants to discuss the issues in the problem – what the error is and why the learner has made that mistake. Also, make sure they have thought of possible strategies to help the learner understand the concepts and how similar mistakes may be avoided.

Take feedback from the groups and discuss the types of error / misconceptions that are found relating to data handling and the best way to deal with them.

Chapter 7
Exemplar schemes of work

7.1 Level 3 Advanced Numeracy for Teachers
Scheme of work for 15 week × 3 hour programme

SCHEME OF WORK				
Course/Subject Level 3 Advanced Numeracy for Teachers			**Lecturer(s)** Daian Marsh/Jackie Ashton	
Unit No/Title ***LSBU Advanced Numeracy for Teachers***				
Session No.	**Content**	**Outcome** at the end of the session learners will be able to	**Assessment Techniques**	**Resources**
1	• *Course induction* • *Level 3 carousel* • *Learning styles*	• *Familiarise themselves with the content of the course handbook and the course requirements* • *Identify areas of strengths and weaknesses with respect to personal numeracy skills* • *Use Gregorc learning style to discover own preferred learning style.*	• *Observation of course handbook activity* • *Self-assessment activity* • *Tactile activity and discussion* • *Group activities* • *Set unit assessment (part 2)*	• *Course guides incorporating unit guides.* • *Level 3 carousel and checklist* • *Gregorc cards and information*

2	• *History of number* • *Percentages* • *Standard form* • *ILPs* • *Learner errors*	• *Identified the importance of the denary place value system* • *Investigate an approach to percentage calculation* • *Perform standard form calculations* • *Identify personal strengths and areas for development* • *Identify common causes of learner errors relating to Number –whole number questions*	• *Group activities and discussions* • *Individual exercises* • *Individual exercise* • *Group activities*	• *PowerPoint and DVD* The Story of Maths – The Language of the Universe(Time:06:05 – 08:09 and 08:35 -10:14) • *Level 3 exam questions incorporating percentages and standard form* • *Learner errors PowerPoint and questions*
3	• *Mathematical models* • *Area and volume* • *Learner errors*	• *Create models for a division of a fraction by a fraction* • *Revise area and volume and solve problems associated with them* • *Identify common causes of learner errors relating to Number FDP questions*	• *Group work activity and flipchart presentations of models* • *Group and individuals tasks and exercises* • *Group activities*	• *PowerPoint* • *Flip chart paper and pens* • *Variety of numeracy resources* • *Learner errors questions*

4	• *Ratio and trigonometry* • *Pythagoras' theorem* • *Learner errors*	• *Recognise how the three trigonometry ratios are derived* • *Use Pythagoras' theorem and trigonometry to solve problems* • *Revise ratio and scale and solve problems associated with them* • *Identify common causes of learner errors relating to MSS questions*	• *Group task and discussion.* • *Individual exercise. Homework exam paper practice* • *Group tasks and exercises.* • *Group activities*	• *Rulers, protractors* • *Scientific calculators PowerPoint* • *Trig handouts and questions* • The Story of Maths – The Language of the Universe(Time 29:00 - 31:33) • *Pythagoras' PowerPoint, handouts and questions* • *Learner errors exam questions*
5	• *Presentation of higher maths skills*		• *Unit assessment (part 2)*	
6	• *Probability* • *Tutorials*	• *Identify and apply concepts and calculations associated with probability and simple and combined events.* • *Calculate probability using tree diagrams and sample space diagrams.* • *Distinguish between permutations and combinations*	• *Observation and discussion of probability scale activity and exercise in pack* • *Individual written exercises* • *Group work and discussion*	• *Cards and scales* • *Probability pack*

7	• *Introduction* • *Carousel* • *Types of data* • *Tutorials* • *Learner errors*	• *Understand and use a range of charts and tables* • *Identify a range of sampling techniques and identify whether sampling methods used are fair and reliable.* • *Identify common causes of learner errors relating to data handling questions*	• *Observation of carousel activities* • *Self assessment on carousel activities* • *Completion of data classifying and learner errors tasks* • *Discussions on data, sampling and learner errors* • *Group activities*	• *Attribute cards* • *Blu-tack* • *Data type card* • *Data collection sheet* • *Type of data handout* • *Sampling handout* • *Data handling carousel activities* • *Laptops* • *Data handling questions*
8	• *Location and dispersion*	• *Interpret the use of statistical techniques such as measures of location and dispersion within reports.* • *Understand and use a range of statistical terms including statistically significant* • *Represent information in the most appropriate format, using IT where possible.*	• *Observation of activities* • *Completion of location/ dispersion matching task* • *Completed charts and tables* • *Discussions on statistical significance and appropriate techniques*	• *Tape measures, flipchart, IWB* • *Calculators* • *Location / dispersion cards* • *Handout on Normal distribution* • *Computers*

9	• *Regression and correlation*	• *Calculate Pearson's correlation coefficient* • *Identify and discuss strength of correlation* • *Structure a report*	• *Completed correlation task* • *Discussion on relationship between variables* • *Set Unit assessment (part 1)*	• *Matching correlation cards* • *Laminated graph paper* • *Non-perm pens* • *Handout on regression and correlation* • *Computers*
10	• *Error analysis* • *Workshop*		• *Unit assessment (part 4ii)*	
11	• *Introduction* • *Carousel*	• *Identify and review their personal skills and knowledge with respect to algebra* • *Substitute values into expressions and formulae* • *Write expressions and equations*	• *Self-assessment of carousel of activities and written exercises.* • *Observation of whole group starter* • *Pair and group activity*	• *Carousel activities* • *Answers* • *Self-assessment checklist* • *Algebra cards* • The Story of Maths – Genius of the East (Time 39:18 – 44:21) • *Handouts and worksheets on fundamental algebra* • *Mini whiteboards*

12	• *Expressions and equations* • *Straight line graphs*	• *Define the term linear in context* • *Draw straight line graphs using the equation and compare with other methods for drawing straight line graphs* • *Identify key characteristics of a straight line graph and match them to parts of the equation y = mx + c* • *Solve linear equations*	• *Discussion and questions* • *Card matching activity and discussion* • *Small group activity on conversion graph drawing* • *Excel spreadsheet activity* • *Written exercises and homework*	• *Excel spreadsheet* • *Expression cards* • *Equation matching cards* • *Graph paper* • *Handouts*
13	• *Simultaneous equations* • *Mock assessment*	• *Solve simultaneous equations*	• *Hand in Unit assessment (part 1)* • *Written mock test*	• *Handouts* • The Story of Maths – Genius of the East (Time 11:17 - 13:19) • *Laptops* • *Mock assessment*
14	• *Written assessment tasks*		• *Unit assessment (part 4 i and ii)*	
15	• *Group assessment* • *Exit tutorials*		• *Unit assessment (part 3)*	

7.2 Level 3 Advanced Numeracy for Teachers
Scheme of work for 5 day × 5 hour programme

SCHEME OF WORK				
Course/Subject Level 3 Advanced Numeracy for Teachers			**Lecturer(s)** Daian Marsh/Jackie Ashton	
Unit No/Title ***LSBU Advanced Numeracy for Teachers***				
Day No.	**Content**	**Outcome** at the end of the session learners will be able to	**Assessment Techniques**	**Resources**
1 am	• *Course induction* • *Level 3 carousel* • *Learning styles* • *Learner errors*	• *Familiarise themselves with the content of the course handbook and the course requirements* • *Identify areas of strengths and weaknesses with respect to personal numeracy skills* • *Use Gregorc learning style to discover own preferred learning style.* • *Identify common causes of learner errors relating to Number questions*	• *Observation of course handbook activity* • *Self-assessment activity* • *Tactile activity and discussion* • *Group activities* • *Set unit assessment (part 2)*	• *Course guides incorporating unit guides.* • *Level 3 carousel and checklist* • *Gregorc cards and information* • *Learner errors PowerPoint and questions*
pm	• *Percentages* • *Standard form* • *Area and volume* • *ILPs*	• *Investigate an approach to percentage calculation* • *Perform standard form calculations* • *Revise area and volume and solve problems associated with them* • *Identify personal strengths and areas for development*	• *Written exercises and discussions* • *Individual exercises* • *Group and individuals tasks and exercises* • *Individual goal setting*	• *PowerPoint* • *Level 3 exam questions incorporating percentages and standard form* • *ILPs*

2 am	• *Location and dispersion* • *Learner errors*	• *Represent information in the most appropriate format, using IT where possible.* • *Interpret the use of statistical techniques such as measures of location and dispersion within reports* • *Identify common causes of learner errors relating to data handling questions*	• *Observation of activities* • *Group exercises* • *Group exercise*	• *Tape measures, flipcharts, IWB* • *Calculators* • *Location/dispersion cards* • *Computers* • *Learner errors handout*
pm	• *Regression and correlation*	• *Understand and use a range of charts and tables* • *Calculate Pearson's correlation coefficient* • *Identify a range of sampling techniques and identify whether sampling methods used are fair and reliable.*	• *Observation of group matching location/dispersion activity* • *Observation of group matching correlation activity* • *Set Unit assessment (part 1)*	• *Matching correlation cards* • *Laminated graph paper* • *Non-perm pens*
3 am	• *Expressions and equations* • *Simultaneous equations*	• *Substitute values into expressions and formulae* • *Write expressions and equations* • *Solve simultaneous equations*	• *Observation of whole group starter* • *Pair and group activity* • *Individual exercise*	• *Algebra cards* • *Expression cards* • *Handout on simultaneous equations*

pm	• *Trigonometry* • *Pythagoras' theorem*	• *Recognise how the three trigonometry ratios are derived* • *Use Pythagoras' theorem and trigonometry to solve problems*	• *Group task and discussion* • *Individual exercise* • *Homework exam paper practice* • *Group tasks and exercises*	• *Rulers, protractors* • *Scientific calculators PowerPoint* • The Story of Maths – The Language of the Universe(Time 29:00 - 31:33 • *Trig handouts and questions* • *Pythagoras' handouts and questions*
4 am	• *Tutorials* • *Workshop*			• *Exam papers*
pm	• *Presentations*		• *Unit assessment (part 2)*	• *Computers for PowerPoint presentations*
5 am	• *Error analysis* • *Writing task* • *Personal maths skills test*		• *Unit assessment (part 4)*	
pm	• *Group task* • *Summary and evaluation*		• *Unit assessment (part 3)*	

Chapter 8
Recommended resources

8.1 Recommended resources

Books and journals	Section
Haylock, D. (2001) *Mathematics Explained.* Paul Chapman Publishing	Number
Liping, M. (1999) *Knowing and Teaching Elementary Mathematics*. New York:Lawrence Erlbaum Associates Publisher	Number
Polya, G. (1990) *How to solve it : a new aspect of mathematical method*. London: Penguin	Number
Code Breakers Level A by Green Board Games- ISBN 1-933054-31-X Code Breakers Level B by Green Board Games - ISBN 1-933054-32-8	Algebra
DfES (2003) *National Needs and Impact Survey of Literacy Numeracy and ICT Skills.* October 2003. London : DfES	Handling data
Rowntree, D. (1981) *Statistics without tears – an introduction for non-mathematicians.* London: Penguin.	Handling data
Miller, J. (2002) *Mathematics for the Future.* Cheltenham: Nelson-Thornes	All
Greer, A. (1992) *A Complete GCSE Mathematics, Higher Course*. Cheltenham: Nelson-Thornes	All
Clapham, C. & Nicholson, J. (2005) Third edition *Oxford Concise Dictionary of Mathematics.* Oxford, Oxford University Press.	All

Websites	Section
Entry assessment requirement http://www.lluk.org/documents/new_entry_guidance.pdf	Assessment
http://people.usd.edu/~ssanto/gregorc.html (accessed 02/10/09)	Learning Styles
AQA Exam papers http://web.aqa.org.uk/qual/elc/numeracy_assess.php	Error analysis
http://www.eyelid.co.uk/numbers.htm (accessed 02/10/09)	Number
http://www.math.wichita.edu/history/topics/num-sys.html#babylonian (accessed 02/10/09)	Number
http://gwydir.demon.co.uk/jo/numbers/babylon/index.htm (accessed 02/10/09)	Number
http://en.wikipedia.org/wiki/Roman_numeral (accessed 02/10/09)	Number
http://www.dfes.gov.uk/research/data/uploadfiles/RR490.pdf (accessed 02/10/09)	Handling data
http://www.literacytrust.org.uk/Database/basicskillsupdate.html#long (accessed 02/10/09)	Handling data
http://nces.ed.gov/nceskids/probability/probability.asp (accessed 29/10/09)	Handling data
http://www.bbc.co.uk/skillswise/numbers/handlingdata/probability/ (accessed 29/10/09)	Handling data
http://www.mathgoodies.com/lessons/vol6/intro_probability.html (accessed 29/10/09)	Handling data
http://www.stats.gla.ac.uk/steps/glossary/probability.html (accessed 29/10/09)	Handling data
http://www.mathsrevision.net/gcse/pages.php?page=32 (accessed 29/10/09)	Handling data
http://stattrek.com/AP-Statistics-2/Survey-Sampling-Bias.aspx (accessed 29/10/09)	Handling data

http://davidmlane.com/hyperstat/ (accessed 29/10/09)	Handling data
https://www.cia.gov/library/publications/the-world-factbook/ (accessed 29/10/09)	Handling data – summative assessment
http://en.wikipedia.org/wiki/Pythagorean_theorem (accessed 02/10/09)	Geometry
http://www.ies.co.jp/math/java/samples/pytha2.html(accessed 02/10/09)	Geometry
http://en.wikipedia.org/wiki/Pythagorean_theorem(accessed 02/10/09)	Geometry
Key Skills Application of Number Level 3 exam papers http://www.ocr.org.uk/qualifications/type/ks/maths/app_number/documents/index.aspx	Summative assessments

Materials	**Section**
Geometric shapes – GLS Supplies http://www.glsed.co.uk/productInfo.aspx?id=-1&tier1=Numeracy&tier2=Shape+%26+Colour&catRef=424469	Level 3 assessment
Dice – GLS supplies http://www.glsed.co.uk/productinfo.aspx?kw=dice&catref=478068 Taskmaster - http://www.taskmasteronline.co.uk/prod-detail.asp?ProductID=2484	Handling data
The Story of Maths DVD The Open University www.ouw.co.uk Available from Amazon http://www.amazon.co.uk/Story-Maths-DVD-Marcus-Sautoy/dp/B001M48UTG/ref=sr_1_1?ie=UTF8&s=dvd&qid=1254497145&sr=8-1	Number, geometry and algebra

Chapter 9
Exemplar entry assessment resources

9.1 Assessment: Group task

Task	Group Task: Gift wrapping task

Gift wrapping Task – Instructions to assessors:

You will need to provide calculators, wrapping paper, scissors, rulers, sticky tape, and access to Excel.

Candidates should be placed in small groups (ideally no more than 4 in a group) and be asked to take part in a group task based on algebra.

It is a discussion based task and you will need to emphasise that every group member will need to participate and contribute as they are being assessed against some of the entry criteria.

There will need to be one assessor per group as you will need to assess each group member against the areas in the assessment sheet. You will need to intervene if you feel that one group member is dominating or one member is being reticent about contributing. You may need to prompt them if you feel that their contributions are insufficient to enable you to assess them against the areas on the assessment sheet (alternatively, any gaps could be followed up at interview).

Provide them with the instructions for candidates (see below) and a copy of the assessment sheet and allow them several minutes reading and thinking time.

Give the groups ten minutes to work on the task initially, then provide them with the actual formula and tell them to use the information to adjust their solution if necessary. Give the groups a further ten minutes to work on the task and then ask them to come up with a solution and to feedback on their solution and their approach to solving the problem.

Provide the candidates with the proposed solution to the task and give them five minutes to write a reflection on their involvement in the task and to evaluate the group's solution, suggesting improvements and reasons for differences from the actual solution. Collect these written reflections in to help you assess each candidate.

Criteria References	Group Task: Gift wrapping task

Process skills	Extent
A. **Purpose** :Engage in the solution to a problem using mathematical means	1.1a Recognise situations can be explored beneficially by using mathematics
	1.2a Demonstrate understanding of the purpose and benefits of mathematical modelling
	1.2c Demonstrate understanding of the benefits of identifying and applying the most appropriate and efficient mathematical conceptual knowledge and procedures
	1.2d Demonstrate that making conceptual links between different areas of mathematics and differing mathematical procedures can support mathematical modelling
B. **Reflecting:** With others, suggest appropriate tools and techniques	1.2b Demonstrate understanding of the stages and iterative nature of mathematical modelling including development, trialling, evaluating, amending, applying and representing/displaying
	1.3b Make reasoned selection of tools such as ICT, measuring, calculating and recording equipment
	2.3a Identify variables and their characteristics
C. **Applying 1**: With others, apply appropriate mathematical techniques to solve the problem	1.3a Make reasoned selections of appropriate mathematical procedures
	2.1a Use efficient procedures in familiar situations and coping strategies in unfamiliar settings accepting that change to efficient procedures is necessary for future development
	2.4c Use extended logic and structures when working in multi-step situations
	3.3c Demonstrate logic in choice of appropriate stage of mathematical interrogation and processing to revisit/revise if results obtained are considered to be inappropriate
D. **Applying 2:** With others, adapt the techniques used in the problem solving task where necessary	1.1b Use interrogation/interpretation by asking questions and considering responses. This is in order to negotiate and hence recognise the mathematics within situations
	2.3b Adapt mathematical models to modify/improve the mathematical representation
	2.4a Organise methods and approaches during investigative processes that allow structured development and testing of models and acceptance/rejection of particular methods/operations/tools
	3.3a Test solutions for appropriateness/accuracy via experimentation, inverse operations, alternative methods, comparison
E. **Interpreting**: With others, interpret the mathematical solution and relate back to the given context	2.2a Identify and justify patterns for summarising mathematical situations
	2.3c Use the analysis of pattern to evaluate particular predicted examples of pattern summaries
	3.1a Apply numerical/mathematical solutions to original context
	3.1b Use solutions to inform future mathematical practice
F. **Communicating:** With others, communicate the results of the process in an appropriate way	4.1c Use communication techniques that display accurately the development of mathematical processing and analysis, including multi-step processing
	4.1d Use oral debate appropriately in communicating results
	4.2b Evaluate the clarity of mathematical arguments to self and audience

G. **Evaluating**: With others and individually, evaluate the solution to the mathematical problem	2.4b Collaborate and engage in critical debate as a mechanism for development and testing of logic and structure during processing/ analysis
	4.2a Evaluate efficient/ rigorous and coping strategies, comparing advantages and disadvantages
	4.2c Use self and group reflection as a mechanism to address mathematical efficiency
	4.2d Evaluate impact of conclusions on future investigations

res no.	style	title
05.3	**Assessment sheet**	**Group Task 2: Gift wrapping task**

Name/initials of participant				
A. Engage in the solution to a problem using mathematical means				
B. With others, suggest appropriate tools and techniques				
C. With others, apply appropriate mathematical techniques to solve the problem				
D. With others, adapt the techniques used in the problem solving task where necessary				
E. With others, interpret the mathematical solution and relate back to the given context				
F. With others, communicate the results of the process				
G. With others and individually, evaluate the solution to the mathematical problem				

Key:
Relevant criteria from process skills: ✓ met fully P partially met × not met

Task	Group Task: Gift wrapping task

Instructions to candidates:

This is a discussion-based task and every group member should aim to participate, as you will be assessed on your ability to select and justify procedures. Each group member should be prepared to be involved in feeding back on justifying their group's choice of methods and solution.

You may use a calculator to help you perform the task but you must not access the internet.

- You should identify the different areas of mathematics that are involved in the task.

- It is important to discuss and negotiate which mathematical procedures you are going to use to perform this task. You should consider the advantages and disadvantages of each method proposed by group members. Also consider testing various different procedures and adapting / rejecting them as appropriate.

You will be given further instructions after you have addressed the points above.

Task:
A free newspaper recently reported that Warwick Dumas of the University of Leicester had devised a formula to work out the most efficient amount of paper for wrapping a gift.
They reported the formula as being "$A = 2(ab + ac + bc + c)$
where A is the area of paper needed and a, b and c are the dimensions of the gift". (London Metro 4.12.07)
The newspaper omitted what type of shape Dumas suggested this works for and to give any more details about the dimensions other than that quoted above.

- In your groups, discuss whether you think this formula is correct.
- If you think it is correct, justify how this would work.
- If you think it is not correct, suggest what the correct formula might be.
- Show how you might test the formula to see if it does actually give the most efficient amount of paper needed.

Further instructions / actual formula:
The actual formula Dumas came up with is:
$A = 2(ab + ac + bc + c^2)$
where A is the area of paper needed to wrap a cuboid, a is the longest side and c is the shortest side.
Does this tally with what you came up with?
If not, whose formula is more efficient – yours or that of Dumas?
The website also states that *"In layman's terms, the length of the wrapping paper should be as long as the perimeter of the side of the gift, with no more than 2cm allowed for an overlap. The width should be just a little over the sum of the width and the depth of the gift"*. Are they correct in saying this?

Finally, reflect on your involvement in the task and evaluate your group's solution, identifying any reasons for differences between that and the given solution and suggesting how the approach could have been improved.

London Metro 4.12.07
University of Leicester Press release (4.12.07) (online)
http://www2.le.ac.uk/ebulletin/news/press-releases/2000-2009/2007/12/nparticle.2007-12-04.6745557516 (accessed 06/12/07)

9.2 Assessment: Presentation

Advanced Numeracy for Teachers

Level 3 presentation:

Personal use of higher level mathematics

Task:
Prepare a short (10 minute) presentation where you identify two examples of where you have used mathematics in your life. The two examples should cover a range of the levels of difficulty of mathematics you have had to use, with one example being what you consider to be difficult or higher level maths (see below for some examples of where higher level maths skills might be used), and one example of simple or basic maths. You could have used the maths in your work, previous study or in the home or everyday life.
N.B. If you are already working as a numeracy/maths teacher, please ***do not*** use the topics you teach as examples.
For each example:

a) Briefly outline the situation where you used mathematics, identifying the context clearly.

b) Identify the problem and break it down into stages, identifying the maths/numeracy skills that you needed to solve the problem at each stage.

c) Describe how you solved the problem.

Include any relevant leaflets, documents etc. that relate the situation you are describing. Include a comment on how you might use the mathematical knowledge you have gained in this way to teach maths/numeracy at lower levels.

Your presentation may take any form but you must communicate clearly, both visually and verbally.
You will deliver your presentation to a small group of your peers. The presentation will be followed by a few questions relating to your presentation.
Examples of where you may have used higher level mathematics in your life **(not an exhaustive list)**:
i) Financial mathematics (work, study or home):

Areas of Study	**Sections**	**Examples**
Financial mathematics	Interest	• Compound and Annual Equivalent Rates • Depreciation • Net present values – tables and calculation comparison • Internal rate of return
	Annuities	• Annuities and perpetuities – tables and calculation comparison • Loans and mortgages • Regular payments – with use of geometric progressions
	Time	• Price indices, for example, aggregative and retail price • Time series – additive and multiplicative models, seasonality • Trends and forecasting

ii) Data handling (work or study)

Areas of Study	**Sections**	**Examples**
Collection and display of data	Survey design	• Data sources including use primary and secondary data • Populations, samples and sampling methodology • Questionnaire design • Discrete and continuous data characteristics • Large and raw data sets

	Graphical display	• Standard methods of display and their appropriate selection, comparison and use, for example, histograms, ogives, box and whisker diagrams, probability distributions • Inappropriate display as a mechanism of distortion
Summarising data	Measures of location and dispersion	• Mean, median, mode • Graphical and numeric calculation • Range, semi-interquartile range, deciles • Mean absolute deviation and standard deviation • Coefficient of variation • Continuous and discrete data types • Comparison of use

iii) Maths skills in Computing (work or study)

Areas of Study	**Sections**	**Examples**
Algebra and its application	Vectors and matrices	• Addition, subtraction and multiplication • Transformations, translations, inverses • Determinants • Simultaneous equations
	Logic circuits	• Boolean algebra – zero/unit rules • Logic design and gates • Commutative, distributive associative laws • Boolean expressions for logic circuits

iv) Use of mathematics in previous career e.g. engineering

Areas of Study	**Sections**	**Examples**
Trigonometry	Ratios, measures and techniques	• Sine, cosine, tangent, radian measure • Cartesian and polar coordinates • Solution of triangles, including sine, cosine rules and area of triangle • Vector force systems
	Functions and graphs	• Nature and graphs of oscillatory functions • Periodic times, frequency and amplitude • Phase difference, angle, harmonics
	Applications	• Metrology/precision measurement, alternating currents, voltages and electrical power, structural design

Advanced numeracy presentation: Higher level maths skills

Record of Assessment

Name of trainee:

Assessor:

Date:

Criterion:	**Comments:**	**Assessment (fully met / partially met / not met) with comments**
Situations that can be analysed and explored through numeracy (1.1)	Trainees should have recognised the numeracy in 2 separate situations: • Basic maths / numeracy • Higher level maths	
The importance of selecting the appropriate numerical information and skills to use (1.4)	Trainees should have identified the appropriate numerical information within the problems and the most appropriate maths/numeracy skills needed to solve the problem – use questioning if not met.	
Methods, operations and tools that can be used in a situation: (1.3)	Trainees should have identified which mathematical strategies / methods to use in each situation.	
Criterion:	**Comments:**	**Assessment (fully met / partially met / not met) with comments**
The importance of using appropriate procedures	Trainees should have used an appropriate mathematical procedure to solve the problems – use questioning to follow up	

and the need to reflect on any process to consider whether other approaches would have been more effective (2.1 & 4.2)	on the importance of this.	
The importance of choosing appropriate language and forms of presentation to communicate results (4.1)	The presentation should be carried out using appropriate, clear language and visual aids.	

9.3 Assessment: Statistics assignment

Name of trainee	
Title of course / qualification	Advanced Numeracy for Teachers
Title of unit(s)	Advanced Numeracy for Teachers Statistical Investigation
Reference and title of assignment	LLU_0_ANT
Date of submission	

a. Read the PISA report http://www.nfer.ac.uk/publications/other-publications/downloadable-reports/pisa-2006.cfm and comment on the sampling methods used with reference to reliability and fairness. Include a discussion on possible sampling errors and possible sources of bias.
b. Prepare a report, with appropriate statistical diagrams* and conclusions, on the key findings of the PISA report. Focus your report **only** on the results for the following countries: Finland, France, Mexico, New Zealand, United States and UK. Results are given for mean scores, percentage of students at different proficiency levels, standard deviation and percentiles; comment on which of these results gives the most accurate picture of numeracy levels in the different countries and why.
c. Explain, in your own words, why Slovenia performed significantly better than the UK while there was no significant difference between Germany and the UK, even though Slovenia and Germany had the same mean score.
d. Look at the statistical data for France and select 2 different facts that you think might be linked to the PISA results.
e. Look at the CIA website: https://www.cia.gov/library/publications/the-world-factbook/ and record the data for the statistical facts you selected in d) for the other five countries.
f. Calculate Pearson's correlation coefficient for the six countries between the mean score from PISA and each of the 2 statistical facts.

*Statistical diagrams should be selected from: Pie charts, Bar charts, Box and Whisker diagrams or any others that are appropriate.

9.4 Assessment: Error analysis task

Name of trainee	
Title of course / qualification	Advanced Numeracy for Teachers
Title of unit(s)	Advanced Numeracy for Teachers Error analysis
Reference and title of assignment	LLU_0_ANT
Date of submission	
Mark (out of 18)	

The following five questions are from assessments by adult students. Each answer is incorrect.
For each question:

a) Write the correct answer,

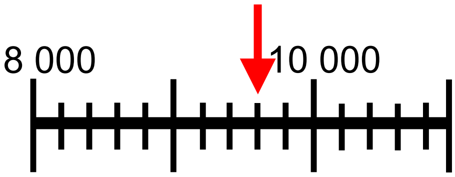
b) Comment on what mistakes you think the student has made

c) Why you think they have made the mistakes

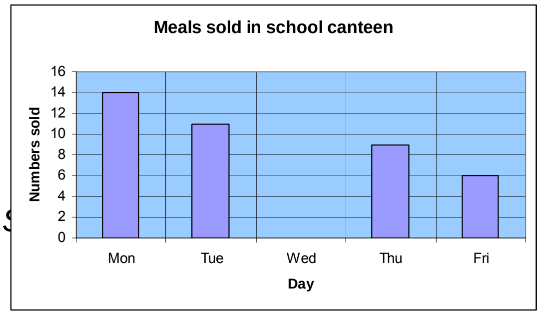
d) Suggest a suitable checking strategy or use of approximation that could be used to check the answer.

We will be awarding 3 marks for each question:

- 1 mark for identifying what mistake the learner has made,
- 1 mark for identifying the mathematical misconception that may have led to the error being made
- 1 mark for suggesting a suitable checking strategy or use of approximation that could be used to check the answer.

Question	Answer / comments
Q1. Subtract 196 from 208 *Student answer* : 208 196 - 192	
Q2. What is the reading on the scale?  *Student answer*: 9 300	
Q3. Round 67 934 to the nearest ten thousand. *Student answer* : 67 000	
Q4. The label on a large bottle of juice states 'dilute 1 part juice to 5 parts water'. How much water must be added to 2 litres of juice? *Student answer*: 2 ½ litres	

Q5. The graph shows the numbers of a particular meal sold in a week in a school canteen. What is the average for the week?



Part 2b Strategies

A learner is struggling with the following question:

> *A man works 8 hours each day. He spends 1 hour each day on paperwork. What percentage of his working day is spent on paperwork?*

Show how you might use visual representation and mathematical equivalences between fractions, decimals and percentages to help the learner solve this problem.

9.5 Assessment: Coverage of areas of maths through assessments

Process skills	Element	Coverage through assessments
1. Making sense of situations and representing them	1.1 Situations that can be analysed and explored through numeracy	Higher level maths presentation, group task, writing task
	1.2 The role of models in representing situations	Group task, writing task
	1.3 Methods, operations and tools that can be used in a situation	Higher level maths presentation, statistics report, group task, maths test, writing task
	1.4 The importance of selecting the appropriate numerical information and skills to use	Higher level maths presentation, statistics report, maths test, writing task
2. Processing and analysis	2.1 The importance of using appropriate procedures	Higher level maths presentation, group task, error analysis, writing task
	2.2 The role of identifying and examining patterns in making sense of relationships (Linear and non-linear situations)	Group task, maths test
	2.3 The role of changing values and assumptions in investigating a situation	Group task, maths test
	2.4 Use of logic and structure when working towards finding results and solutions	Group task, maths test
3. Interpreting and evaluating results	3.1 The role of interpretation of results in drawing conclusions	Statistics report, group task
	3.2 The effect of accuracy on the reliability of findings	Statistics report, maths test
	3.3 The appropriateness and accuracy of results and conclusions	Group task, maths test, error analysis
4.Communicating and reflecting on findings	4.1 The importance of choosing appropriate language and forms of presentation to communicate results	Higher level maths presentation, statistics report, group task
	4.2 The need to reflect on any process to consider whether other approaches would have been more effective	Higher level maths presentation, group task